Alleviating H_0 and σ_8 tensions from WGB topological dark energy

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- Assuming the Hawking entropy $S_h = \frac{A}{4G}$ where $A = 4\pi\tilde{r}_A^2$:

$$-4\pi G(\rho_m + p_m) = \dot{H} - \frac{k}{a^2}, \tag{1}$$

$$\frac{8\pi G\rho_m}{3} = H^2 + \frac{k}{a^2} - \frac{\Lambda}{3} \tag{2}$$

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where h, the BH horizon, is a S^2 sphere, thus its Euler characteristic is $\chi(h) = 2$.

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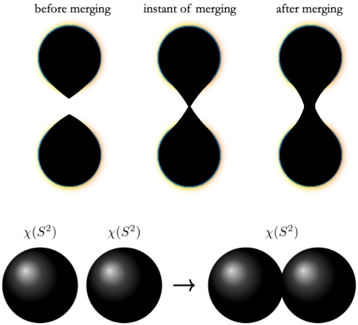

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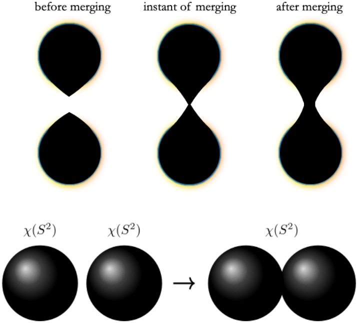

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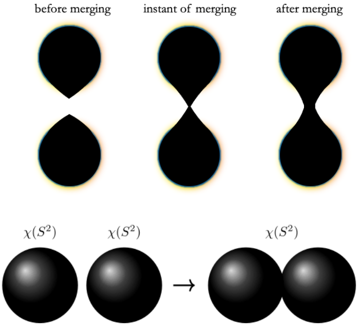

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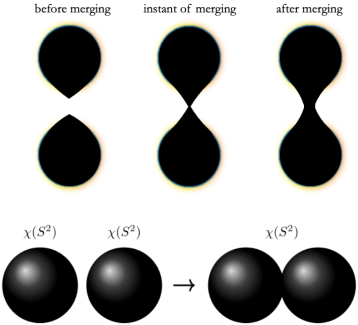

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Vice versa, during BH formation $\delta\chi = 2$

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- By demanding that **the topology of the causally connected boundaries** $\partial M = \mathcal{H} \bigcup_{i=1}^{N} h_i$ **remain constant** $\chi(\mathcal{H}) + \sum_{i=1}^{N} \chi(h_i) = \text{cons.}$

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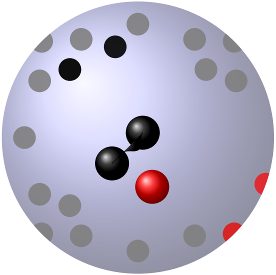

when δN_{form} BHs are formed and δN_{merg} BHs merge

$$\delta\chi(\mathcal{H}) = -2\,(\delta N_{form} - \delta N_{merg})$$

$$\delta\chi(\mathcal{H}) = -2\,\delta N \tag{4}$$

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If instead of the standard Hawking entropy we implement the WGB entropy into the first law $dE = TdS$, follow the steps in the spacetime thermodynamics procedure and transform per redshift, then we obtain:

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$$H^2(z) = \frac{8\pi G}{3}\rho_m + k(1+z)^2 + \frac{\Lambda}{3} - 8\tilde{a}\int_{z_i}^{z}[H^2 + k(1+z)^2]^2 \frac{dN}{dz}dz. \quad (5)$$

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absorbing the extra terms in the DE density we define:

$$\rho_{DE}(z) = \frac{3}{8\pi G}\left\{\frac{\Lambda}{3} - 8\tilde{a}\int_{z_i}^{z}[H^2 + k(1+z)^2]^2\frac{dN}{dz}dz\right\}, \tag{6}$$

Black hole formation and merger rates

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From the Star Formation Rate (SFR) best fit model of Madau and Dickinson [2014]

$$\psi(z) = 0.015 \frac{(1+z)^{2.7}}{1 + \left[(1+z)/2.9\right]^{5.6}} \; M_{\odot} \mathrm{year}^{-1} Mpc^{-3}, \quad (7)$$

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we have estimated the rate of the active number of BHs per redshift inside the apparent horizon

$$\frac{dN(z)}{dz} = C \frac{\psi(z)}{H^4(z)(1+z)}, \quad (8)$$

where we have absorbed all the astrophysical parameters

$$C \equiv \frac{4\pi}{3} \frac{\left(1 - f_{\mathrm{bin}} \times f_{\mathrm{merge}}\right) f_{\mathrm{BH}}}{\langle m_{\mathrm{prog}} \rangle}. \quad (9)$$

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$$w_{DE}(z) = -1 - \frac{2\tilde{\alpha} C \psi(z)}{\frac{\Lambda}{4} - 6\tilde{\alpha} C \int_{z_i}^{z} \frac{\psi(z)}{(1+z)} dz}, \quad (12)$$

Parameters of the model

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The integral can be evaluated in terms of the hypergeometric function ${}_2F_1(a, b; c; z)$, and gives

$$\int \frac{\psi(z)}{(1+z)} dz = 0.37037 \cdot (1+z)^{2.7} \tag{13}$$
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Range of the involved parameters according to the literature.

Parameter	Value
$f_{\rm BH}$	0.1% to 5%
$\langle m_{\rm prog} \rangle$	25 to 40 $M_\odot$
$f_{\rm merge}$	1% to 10%
$f_{\rm bin}$	50% to 80%

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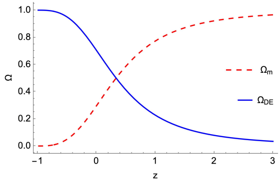

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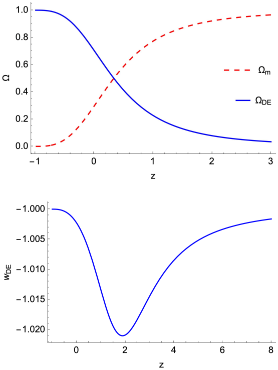

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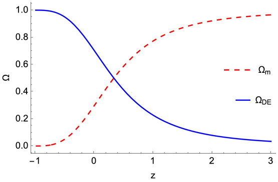

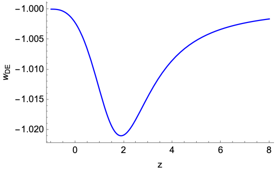

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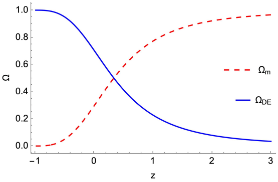

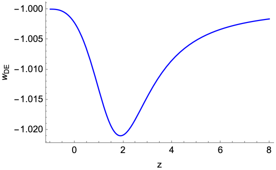

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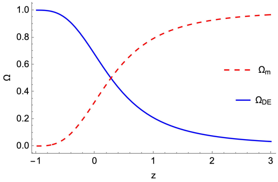

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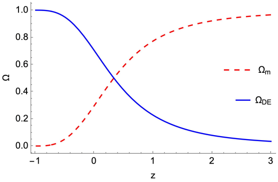

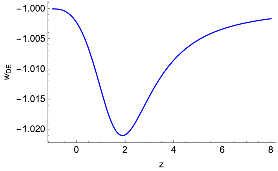

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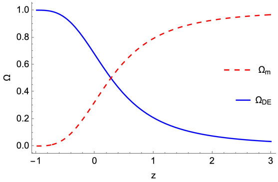

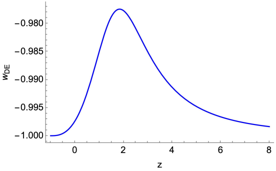

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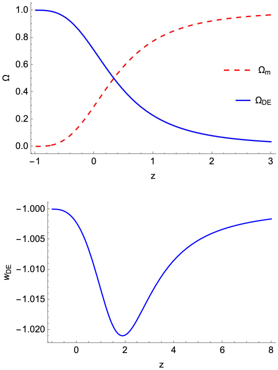

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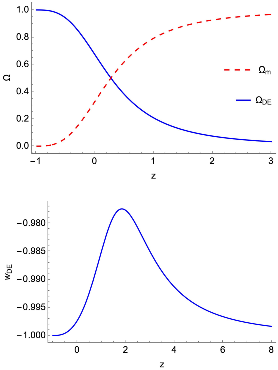

In all graphs we have used the models parameters $|\tilde{\alpha}| = 10^5$ *(in* H_0 *units),* $f_{BH} = 0.025$, $m_{prog} = 30M_{\odot}$, $f_{bin} = 0.65$, $f_{merge} = 0.05$ *and we have implemented* $\Omega_{DE0} = 0.69$.

Alleviating the H_0 tension

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The normalized $H(z)/(1+z^3)$
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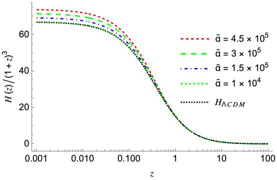

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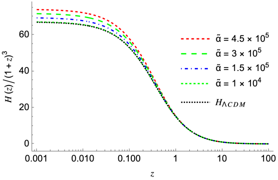

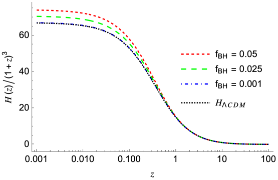

In the upper graph we have set $f_{BH} = 0.025$, while in the lower graph $\tilde{\alpha} = 2.3 \times 10^5$.In both graphs we have used $m_{prog} = 30M_\odot$, $f_{bin} = 0.65$, $f_{merge} = 0.05$ and we have set $\Omega_{m0} = 0.31$.

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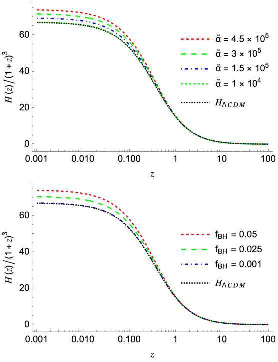

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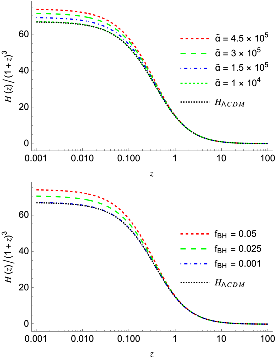

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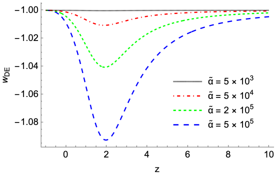

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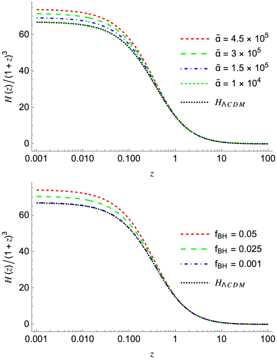

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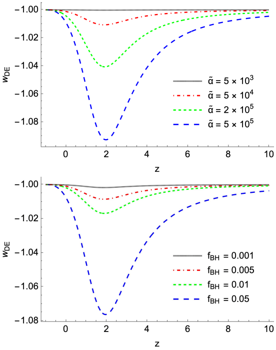

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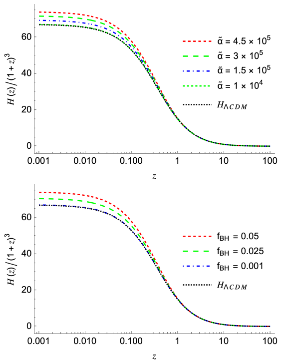

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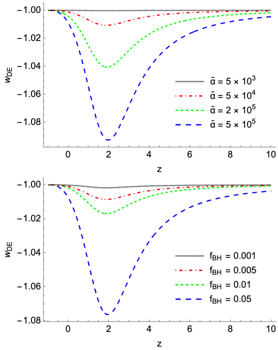

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H_0	75.2 ± 4.7 km/s/Mpc
Ω_{m0}	0.38 ± 0.05
$\tilde{\alpha}$	$(9.4 \pm 7.1) \times 10^4$
Ω_{DE0}	0.62 ± 0.05
r_{drag}	212.1 ± 31.2 Mpc

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$\tilde{\alpha}$	$(9.4 \pm 7.1) \times 10^4$
Ω_{DE0}	0.62 ± 0.05
r_{drag}	212.1 ± 31.2 Mpc

Observational constraints at 1σ confidence level.

We have fixed the astrophysical parameters as: $f_{BH} = 0.025$, $m_{prog} = 30 M_{\odot}$, $f_{bin} = 0.65$, $f_{merge} = 0.05$ so as to constrain the free parameter $\tilde{\alpha}$.

Bayesian analysis of the parameter space

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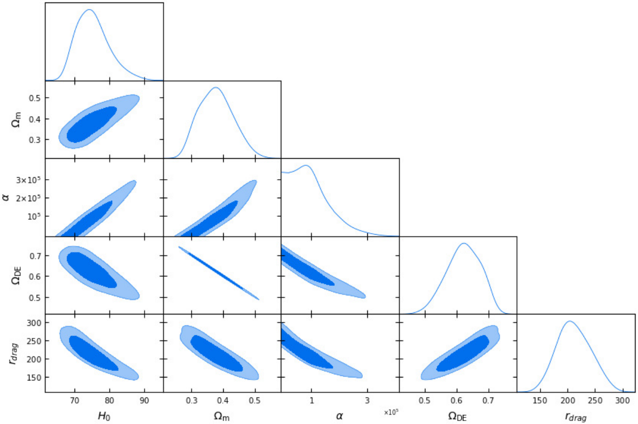

The 1σ and 2σ iso-likelihood contours for Wald-Gauss-Bonnet topological dark energy, for the 2D subsets of the parameter space using Cobaya code.

Alleviating the σ_8 tension

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DE does not cluster, hence modifications on the overdensity evolution will depend mainly on H:

$$\delta_m'' + \left(\frac{H'(z)}{H(z)} - \frac{1}{1+z}\right)\delta_m' - \frac{3\,\Omega m_0\ H_0^2\ (1+z)}{H^2}\delta_m = 0. \tag{14}$$

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after extracting the solution for $\delta_m(z)$ we calculate the important physical observable

$$f\sigma 8 \equiv f(z)\sigma(z), \tag{15}$$

where $f(z) := -\frac{dln\delta_m(z)}{dlnz}$ and $\sigma(z) := \sigma_8 \frac{\delta_m(z)}{\delta_m(0)}$.

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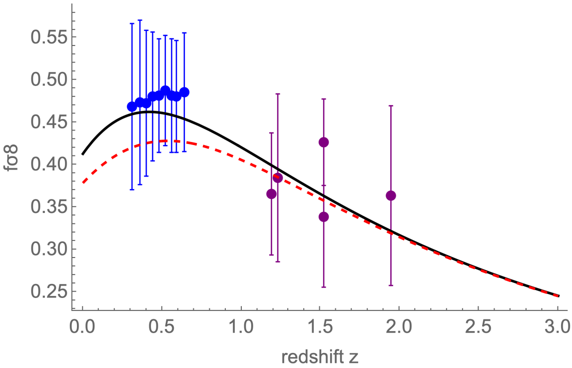

Evolution of $f\sigma 8$ in Wald-Gauss-Bonnet cosmology with $\tilde{\alpha} = 4.5 \cdot 10^5$ in units where $8\pi G = 1$, and with $f_{BH} = 0.025$, $m_{prog} = 30 M_{\odot}$, $f_{bin} = 0.65$, $f_{merge} = 0.05$ (red dashed), as well as in ΛCDM paradigm (black solid). Additionally, the blue data points are from Baryonic Acoustic Oscillations (BAO) observations in SDSS-III DR12 Gil-Marín et al. [2018], while the gray data points at higher redshifts are from SDSS-IV DR14 Hou et al. [2018], Zhao et al. [2019], Gil-Marín et al. [2018].

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Thank you for your attention!!!

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Bibliography I

Rong-Gen Cai and Sang Pyo Kim. First law of thermodynamics and Friedmann equations of Friedmann-Robertson-Walker universe. *JHEP*, 02:050, 2005. doi: 10.1088/1126-6708/2005/02/050.

Héctor Gil-Marín et al. The clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample: structure growth rate measurement from the anisotropic quasar power spectrum in the redshift range $0.8 < z < 2.2$. *Mon. Not. Roy. Astron. Soc.*, 477(2): 1604–1638, 2018. doi: 10.1093/mnras/sty453.

Bibliography II

Jiamin Hou et al. The clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample: anisotropic clustering analysis in configuration-space. *Mon. Not. Roy. Astron. Soc.*, 480(2): 2521–2534, 2018. doi: 10.1093/mnras/sty1984.

Piero Madau and Mark Dickinson. Cosmic Star Formation History. *Ann. Rev. Astron. Astrophys.*, 52:415–486, 2014. doi: 10.1146/annurev-astro-081811-125615.

Sudipta Sarkar and Aron C. Wall. Second Law Violations in Lovelock Gravity for Black Hole Mergers. *Phys. Rev. D*, 83: 124048, 2011. doi: 10.1103/PhysRevD.83.124048.

Jesus Torrado and Antony Lewis. Cobaya: Code for Bayesian Analysis of hierarchical physical models. *JCAP*, 05:057, 2021. doi: 10.1088/1475-7516/2021/05/057.

Bibliography III

Robert M. Wald. Black hole entropy is the Noether charge. *Phys. Rev. D*, 48(8):R3427–R3431, 1993. doi: 10.1103/PhysRevD.48.R3427.

Gong-Bo Zhao et al. The clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample: a tomographic measurement of cosmic structure growth and expansion rate based on optimal redshift weights. *Mon. Not. Roy. Astron. Soc.*, 482(3):3497–3513, 2019. doi: 10.1093/mnras/sty2845.