PRIMORDIAL BLACK HOLES, CHARGE AND DARK MATTER:

RETHINKING EVAPORATION LIMITS

IN COLLABORATION WITH JUSTIN FENG, SEBASTIAN SCHUSTER, AND MATT VISSER

ARXIV: 2503.20696

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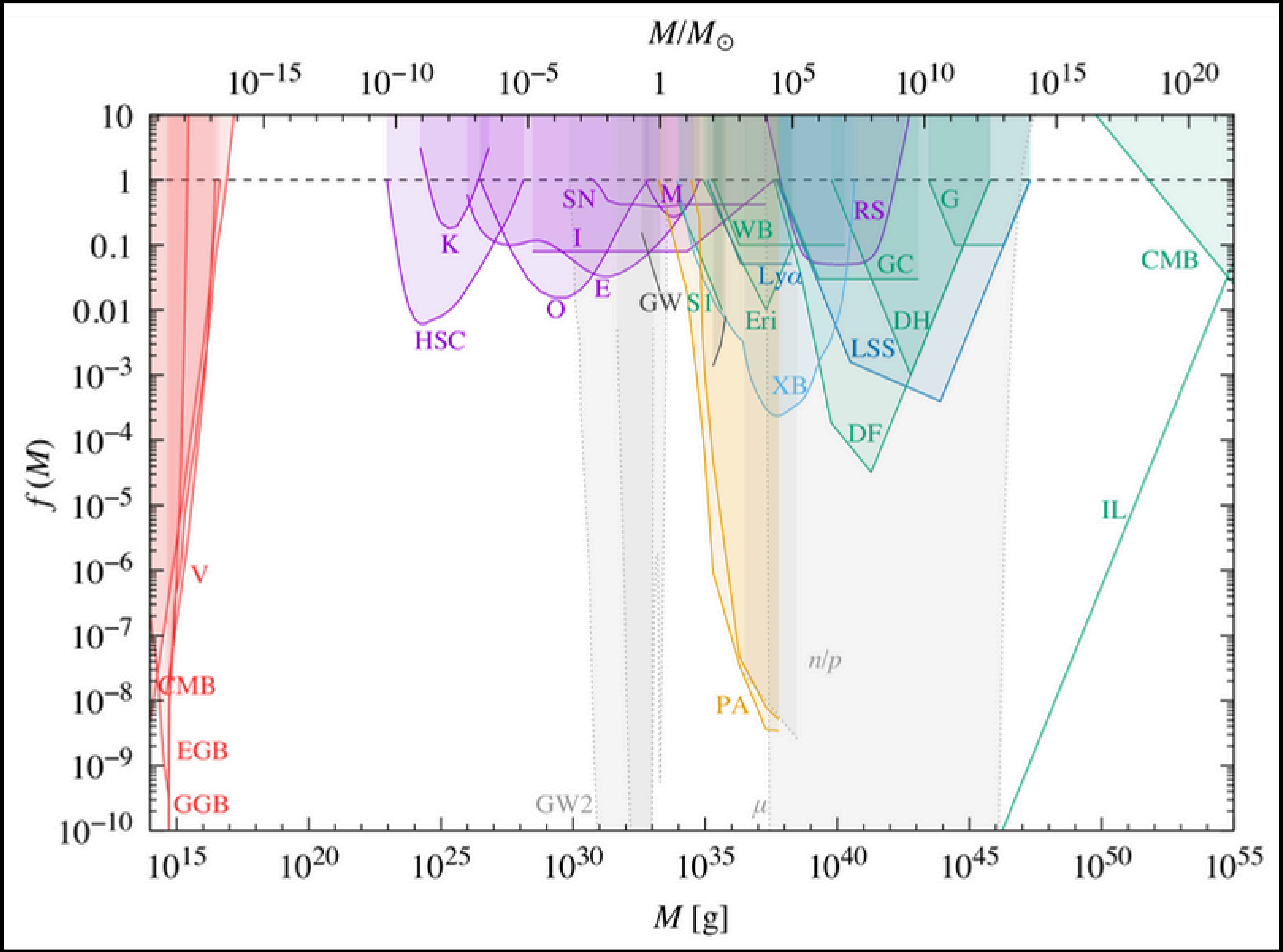
Primordial black holes as DM candidate



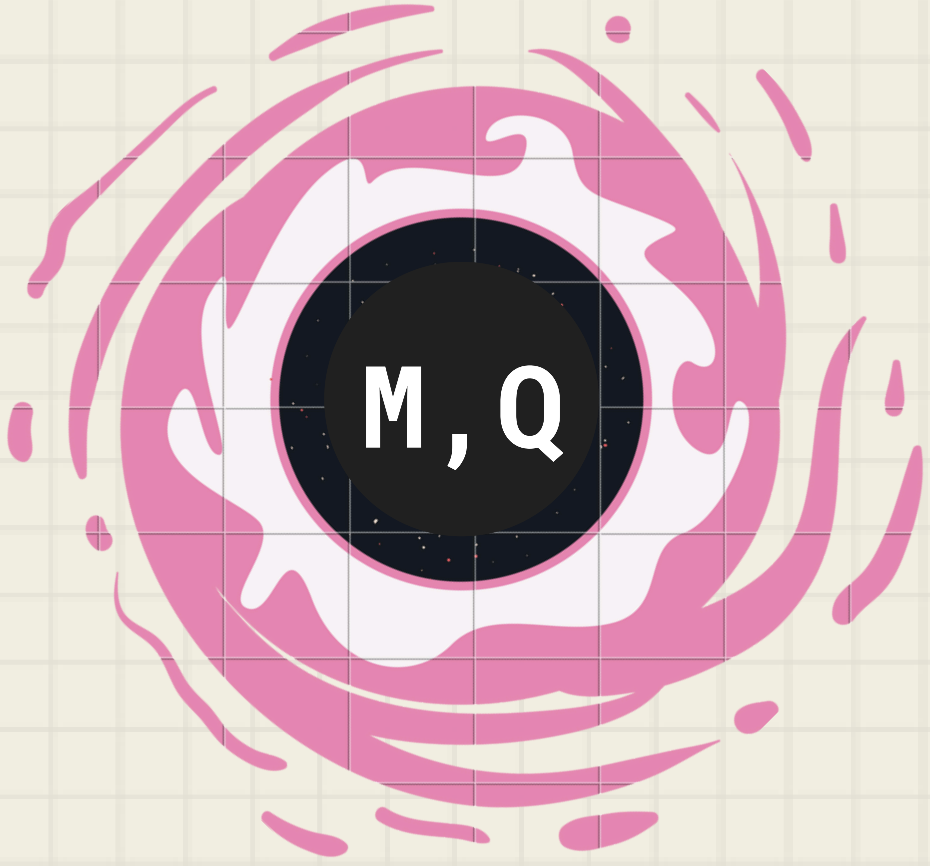
Carr, B., Kohri, K., Sendouda, Y. & Yokoyama, J. (2021)

Reissner Nordstrom metric

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2\, d\Omega^2$$

- It is static and spherically symmetric;
- Solves the coupled Einstein–Maxwell equations
- Has two event-horizons located at:

$$r_\pm = M \pm \sqrt{M^2 - Q^2}$$



The temperature of an evaporating RN black hole is given by:

$$T = \frac{\hbar\kappa}{2\pi}, \qquad \text{with} \qquad \kappa = \frac{(M^2 - Q^2)^{1/2}}{r_+^2}$$

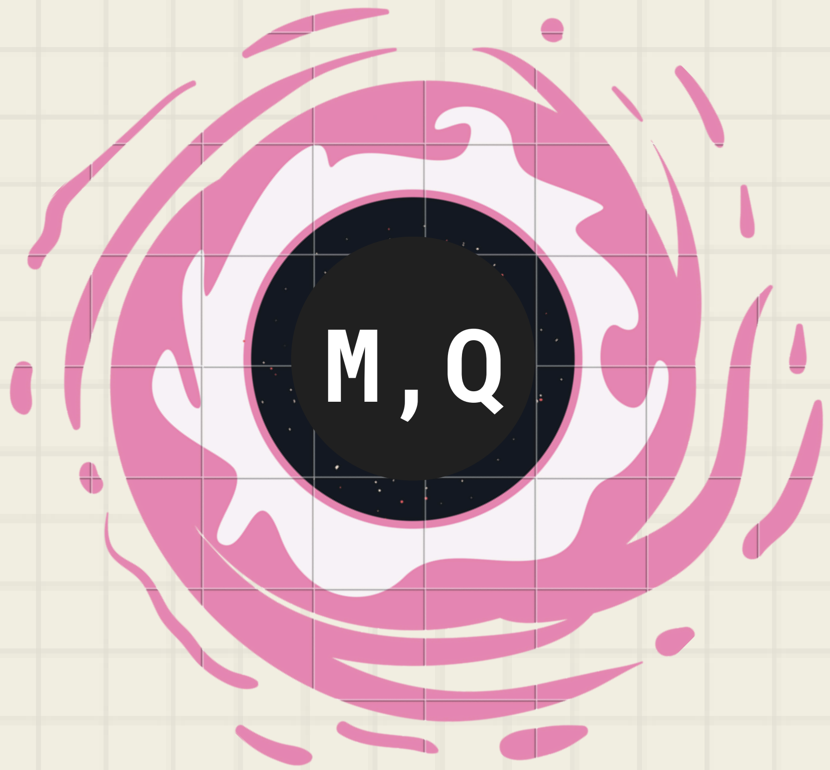
For standard EM:

$$\frac{Q}{M} \ll \frac{m}{e} \simeq 10^{-21} \qquad \text{(in geometric units)}$$

Reissner Nordstrom PBH with dark electric charge

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2\, d\Omega^2$$

$$T = \frac{\hbar\kappa}{2\pi}, \qquad \text{with} \qquad \kappa = \frac{(M^2 - Q^2)^{1/2}}{r_+^2}$$



Dark electromagnetic charge

$$\mathcal{L} = \bar{\chi}(i\not{D} + m_\chi)\chi - \frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}$$

$D_\mu = \partial_\mu - ie_\chi \hat{A}_\mu$ is the gauge covariant derivative

$\hat{F}_{\mu\nu}$ is the field-strength tensor for dark electromagnetism

Ackerman, L., Buckley, M. R., Carroll, S. M. & Kamionkowski, M. (2009)

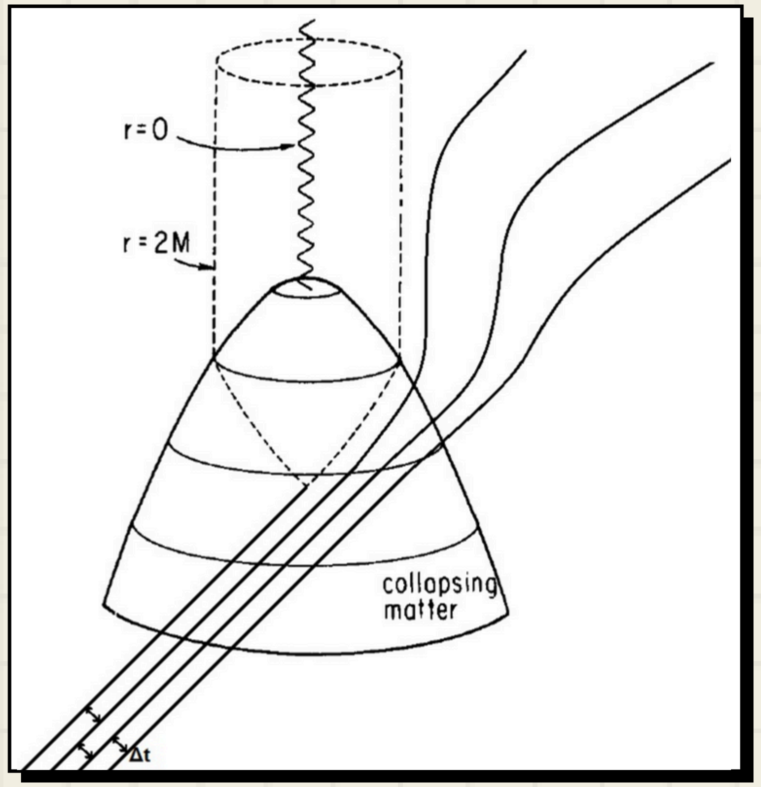
RN BH Evaporation:

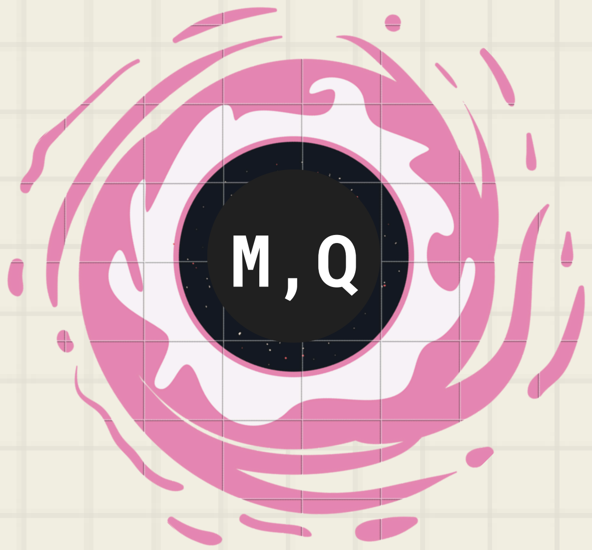
Two effects are responsible:

Hawking effect

Quantum field theory in curved spacetime effect. Is is generated by the presence of an event horizon, which changes the notion of an initial vaccumm state into a thermal bath.

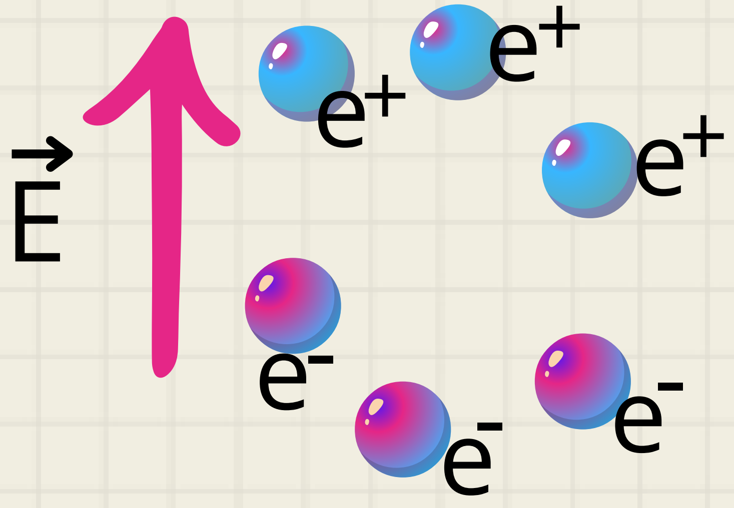
$$\frac{dM}{dt} = -aT^4\alpha\sigma_0$$





Schwinger effect

The presence of a sufficiently strong electric field estimulates the quantum creation of pairs of particles.



The Schwinger expression for the rate of electron-positron pair creation per unit four-volume is given by:

$$\Gamma = \frac{(eE)^2}{4\pi^3\hbar^2 c}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{\pi m_e^2 c^3 n}{\hbar e E}\right)$$

HW assumptions -- Dark EM extension

Positive charge

Applicability of Schwinger's result:

$$\Gamma = \frac{(eE)^2}{4\pi^3\hbar^2 c}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{\pi m_e^2 c^3 n}{\hbar e E}\right)$$

Black hole mass must be much larger than the reduced Compton wavelength of the electron (or lightest charged particle)

$$M \gg \frac{\hbar}{m_e} \simeq 10^{-15} M_\odot$$

Series truncation:

$$\frac{e^3 Q}{m_e^2 r^2} \ll 1$$

Error function series truncation:

$$r_+^2 \gg Q Q_0$$

Summarized model limitations:

$$\text{(i)}\ \ M \gg \frac{\hbar}{m_e}, \qquad \text{(ii)}\ \ \frac{e^3 Q}{m_e^2 r^2} \ll 1, \qquad \text{and} \qquad \text{(iii)}\ \ r_+^2 \gg Q Q_0$$

Dark EM extension:

$$M_{DE} \gg \frac{\hbar}{m_\chi} = \frac{10^{-15}}{\sigma_m} M_\odot$$

$$M_{DE} \gg \frac{e_\chi^3}{m_\chi^2} = 4 \cdot 10^3\, \frac{\sigma_e^3}{\sigma_m^2} M_\odot$$

$$\frac{\sigma_e^2\, \sigma_M}{\sigma_m} \gg 10^{-66}$$

RN BH Evaporation:

The full evolution equations:

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -aT^4\alpha\sigma_0 + \frac{Q}{r_+}\frac{\mathrm{d}Q}{\mathrm{d}t},$$

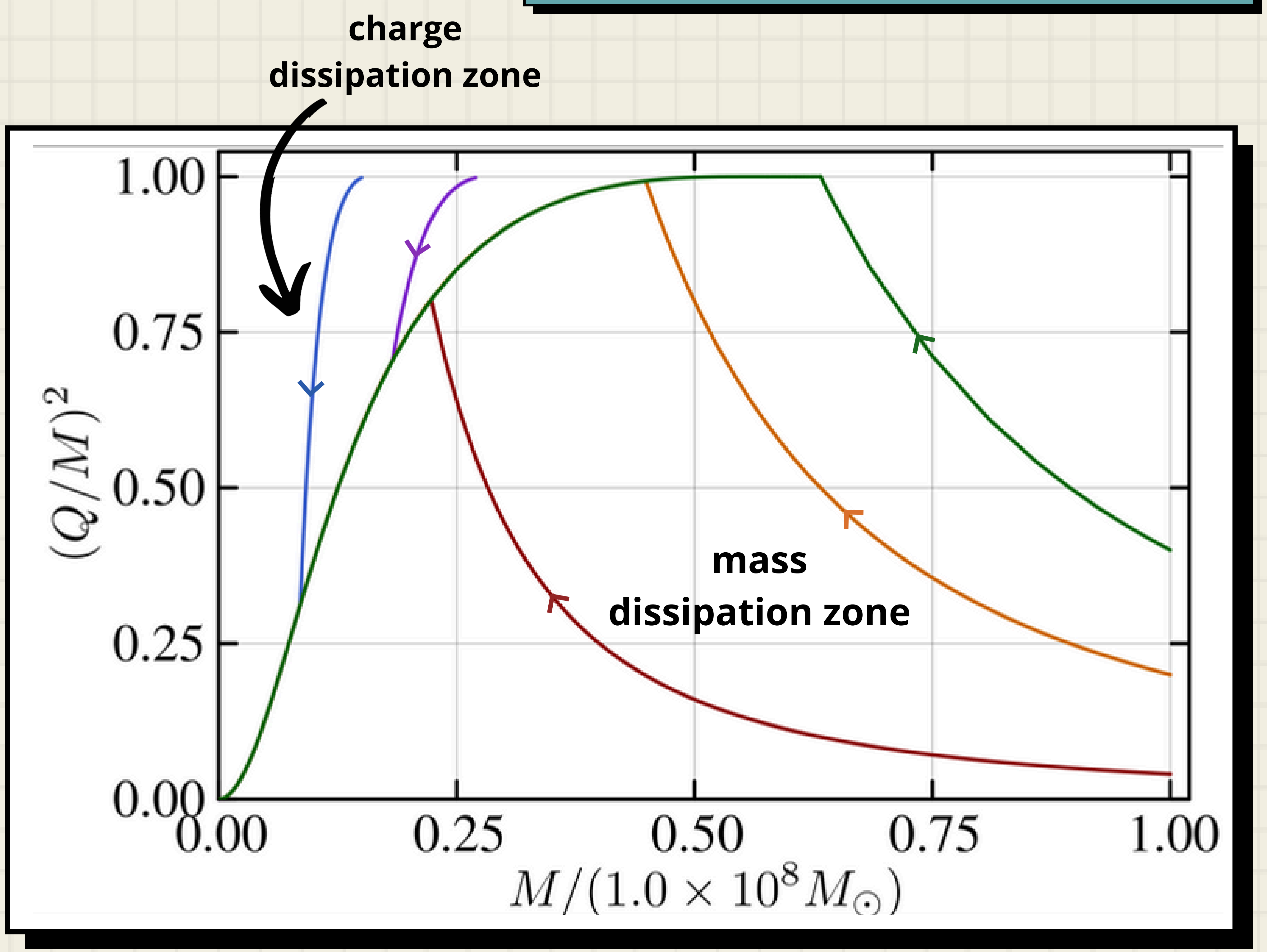
$$\frac{\mathrm{d}Q}{\mathrm{d}t} = -\frac{e^3}{\pi^2\hbar^2 r_+}\exp\left(-\frac{r_+^2}{QQ_0}\right) - \frac{\pi}{\sqrt{QQ_0}}\operatorname{erfc}\left(\frac{r_+}{\sqrt{QQ_0}}\right)$$

$$\sigma_0 := \pi\frac{\left(3M+\sqrt{9M^2-8Q^2}\right)^4}{8\left(3M^2-2Q^2+M\sqrt{9M^2-8Q^2}\right)}$$

$$Q_0 := \frac{\hbar e}{\pi m^2} \approx 1{,}7\cdot 10^5 M_\odot$$

$$a = \pi^2/(15\hbar^3)$$

Hiscock & Weems (1989)



RN BH Evaporation:

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -aT^4\alpha\sigma_0 + \frac{Q}{r_+}\frac{\mathrm{d}Q}{\mathrm{d}t},$$

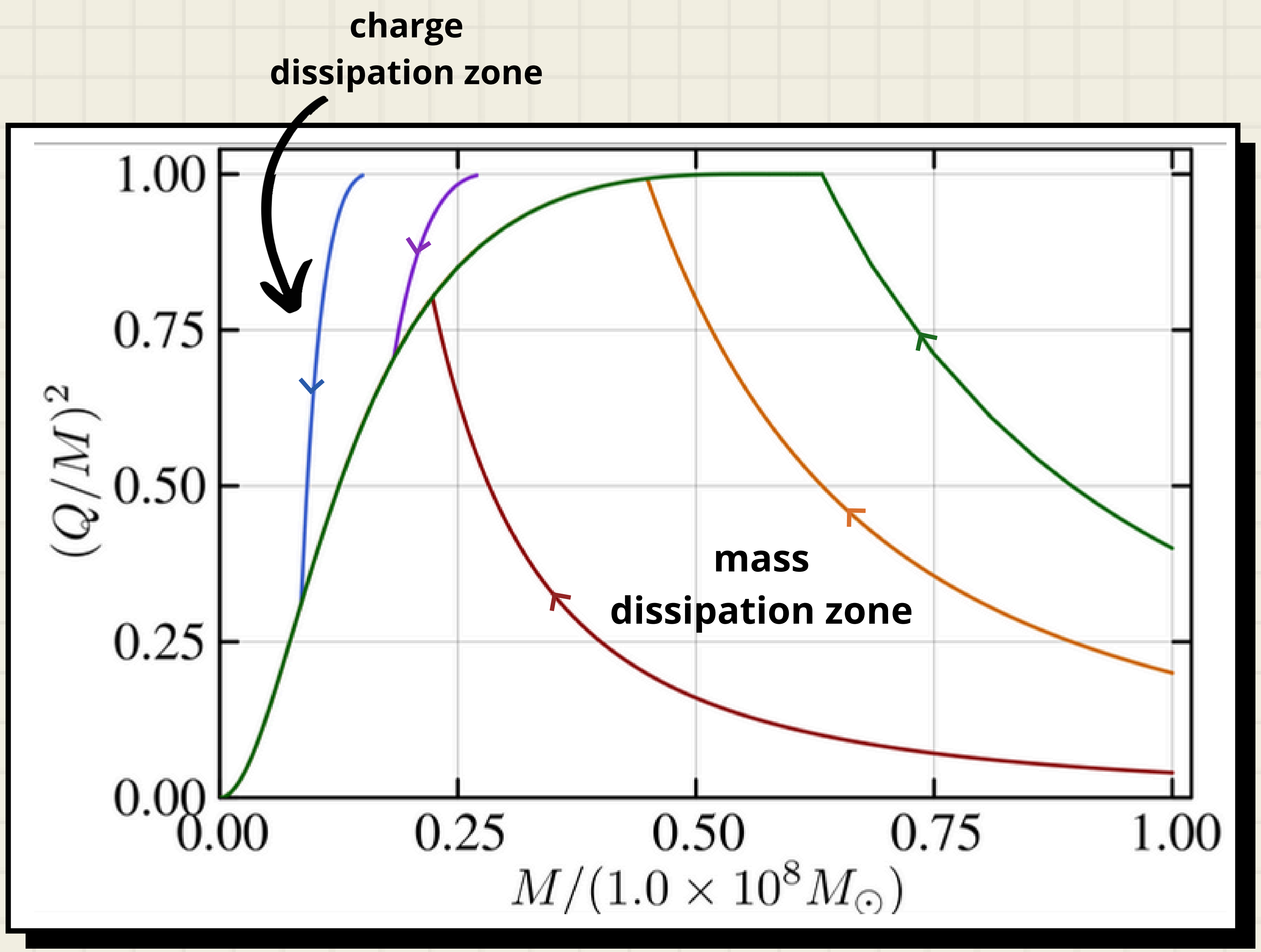
$$\frac{\mathrm{d}Q}{\mathrm{d}t} = -\frac{e^3}{\pi^2\hbar^2 r_+}\exp\left(-\frac{r_+^2}{QQ_0}\right) - \frac{\pi}{\sqrt{QQ_0}}\operatorname{erfc}\left(\frac{r_+}{\sqrt{QQ_0}}\right)$$

change of variables

$$Y := (Q/M)^2 \qquad \mu := M/M_s$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}\tau} = -\frac{(H(\mu,Y) + S(\mu,Y)Y^2)}{(\sqrt{1-Y}+1)^4},$$

$$\frac{\mathrm{d}Y}{\mathrm{d}\tau} = \frac{2\Big(H(\mu,Y) - S(\mu,Y)\Big(1-Y+\sqrt{1-Y}\Big)Y\Big)Y}{\mu(\sqrt{1-Y}+1)^4}$$



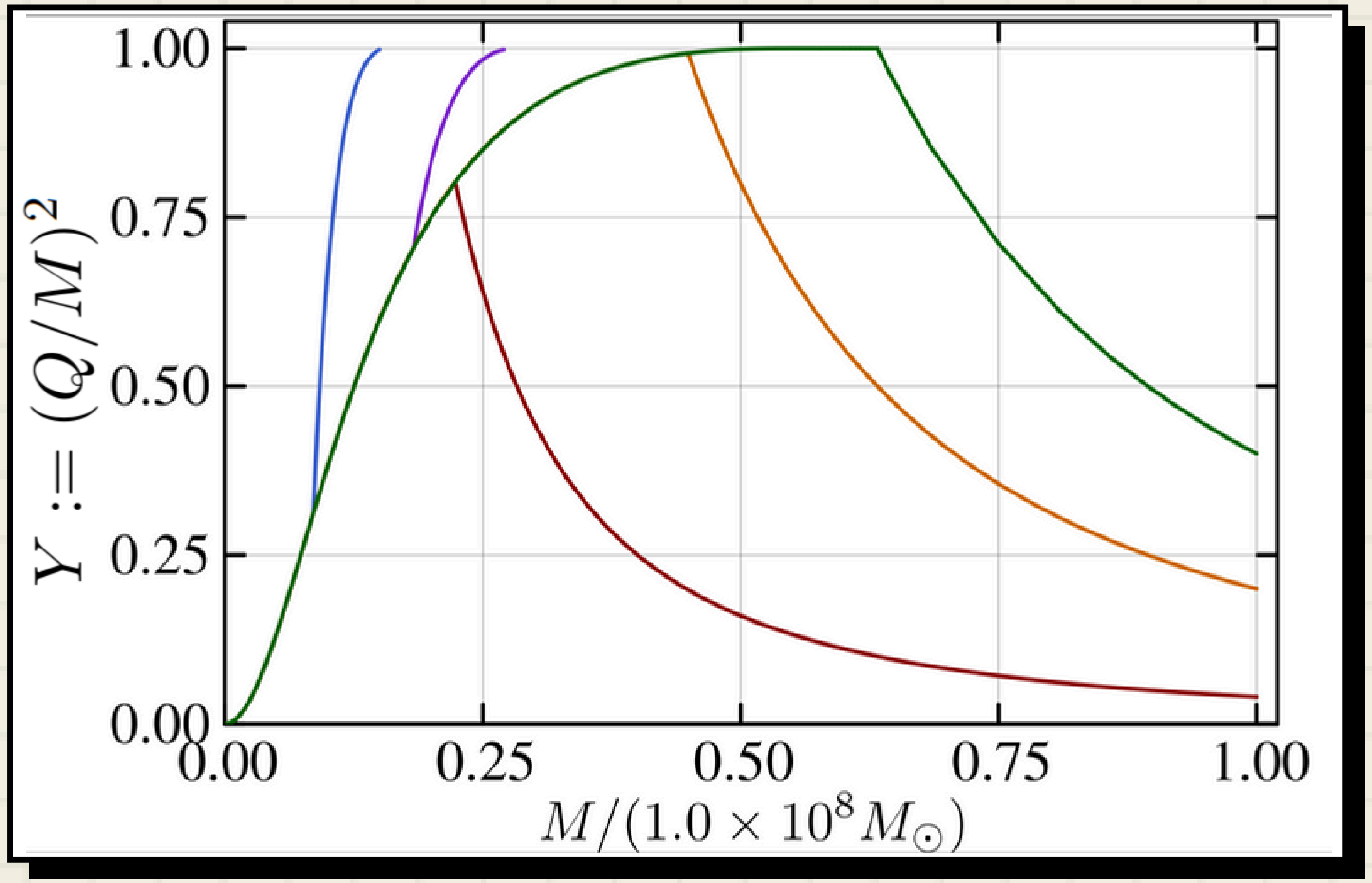
$$H(\mu,Y) := \frac{\left(\sqrt{9-8Y}+3\right)^4(1-Y)^2}{\mu^2(\sqrt{1-Y}+1)^4(3-2Y+\sqrt{9-8Y})},$$ the Hawking term

$$S(\mu,Y) := \exp\left\{b_0\left[z_0 - \mu\left(\sqrt{1-Y}+1\right)^2/\sqrt{Y}\right]\right\}.$$ the Schwinger term

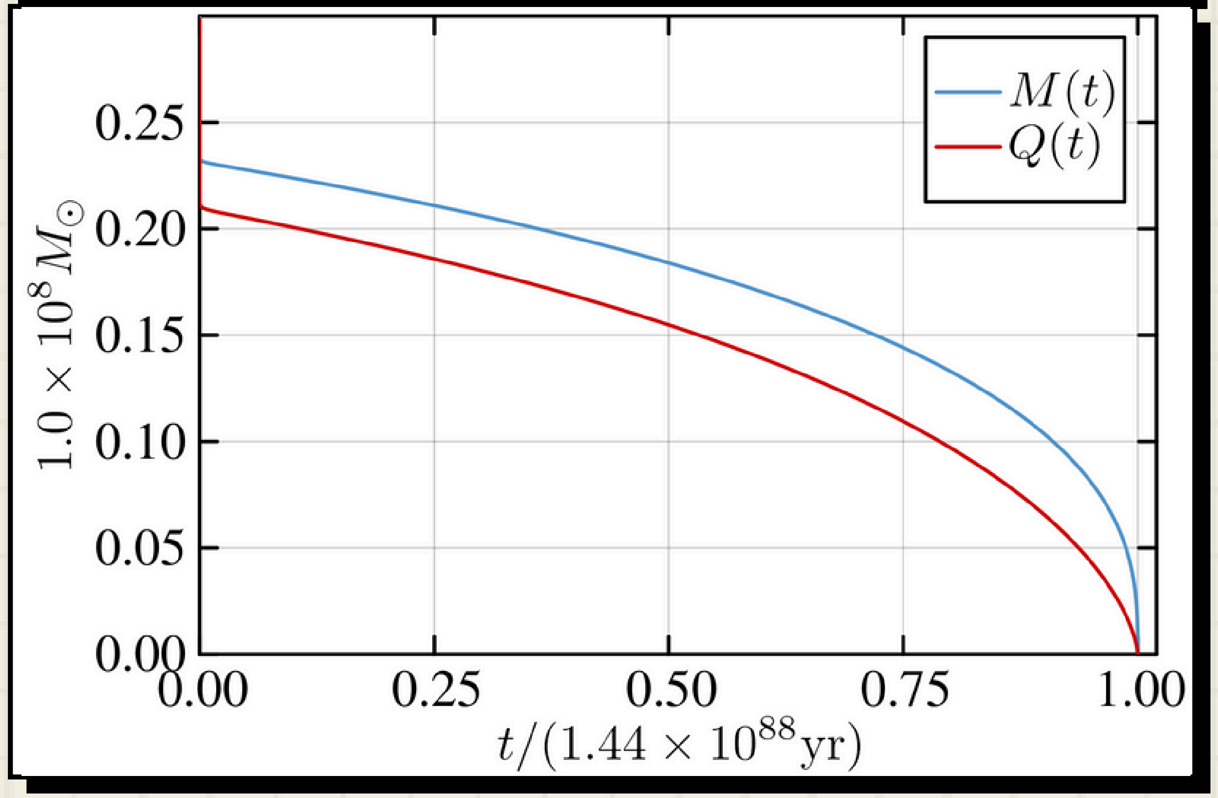
$$z_0 = \frac{e\hbar}{\pi m_e^2 M_s}\ln\left(\frac{960e^4M_s^2}{\pi^2\alpha m_e^2\hbar^2}\right), \qquad b_0 = \frac{m_e^2 M_s \pi}{e\hbar}$$

$$z_0 = 0.53132,\ b_0 = 584.92$$

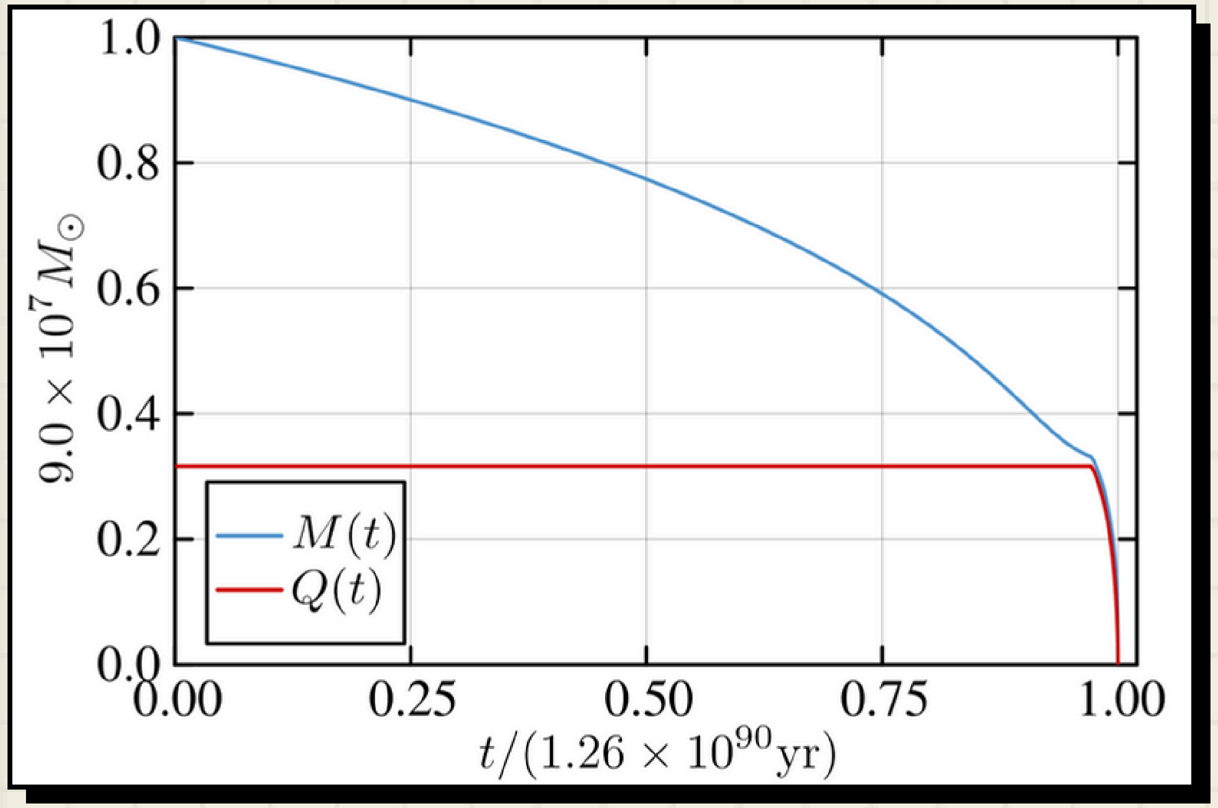
RN BH Evolution Profiles:



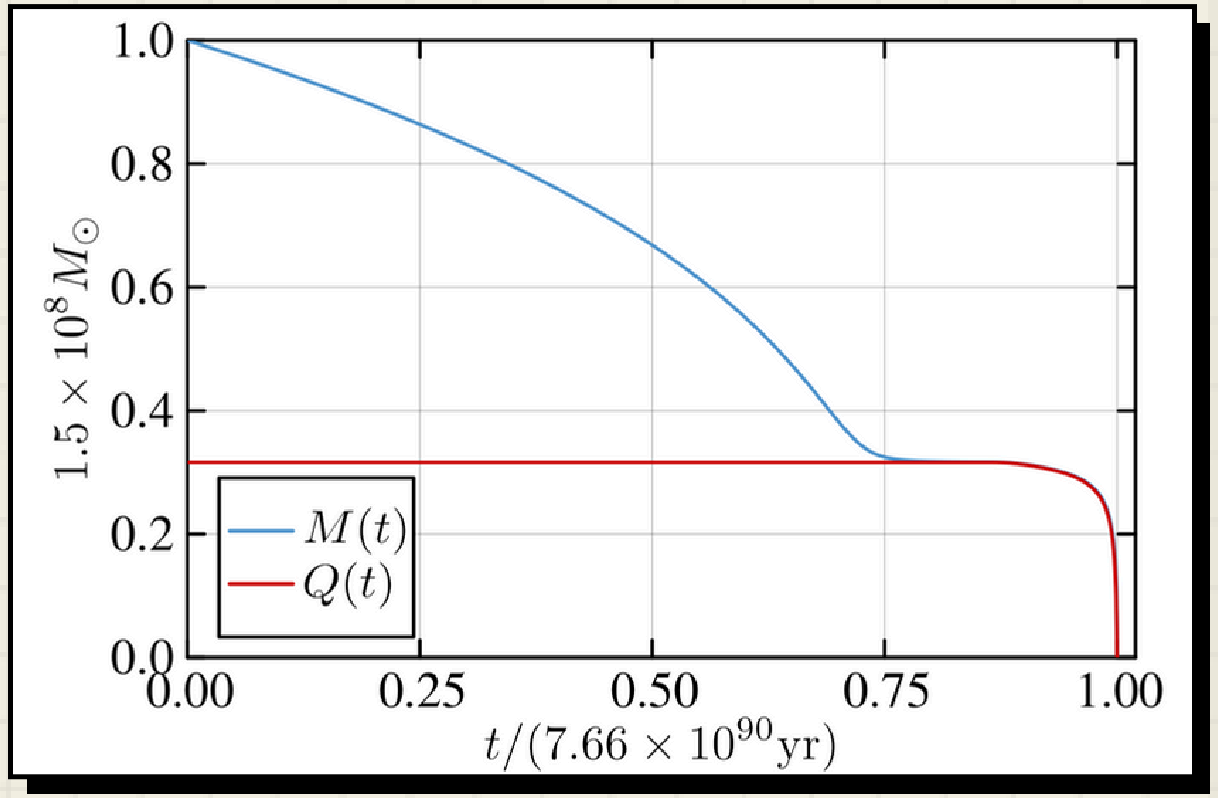
M = $0.3 \times 10^6 M_\odot$, Y = 0.1



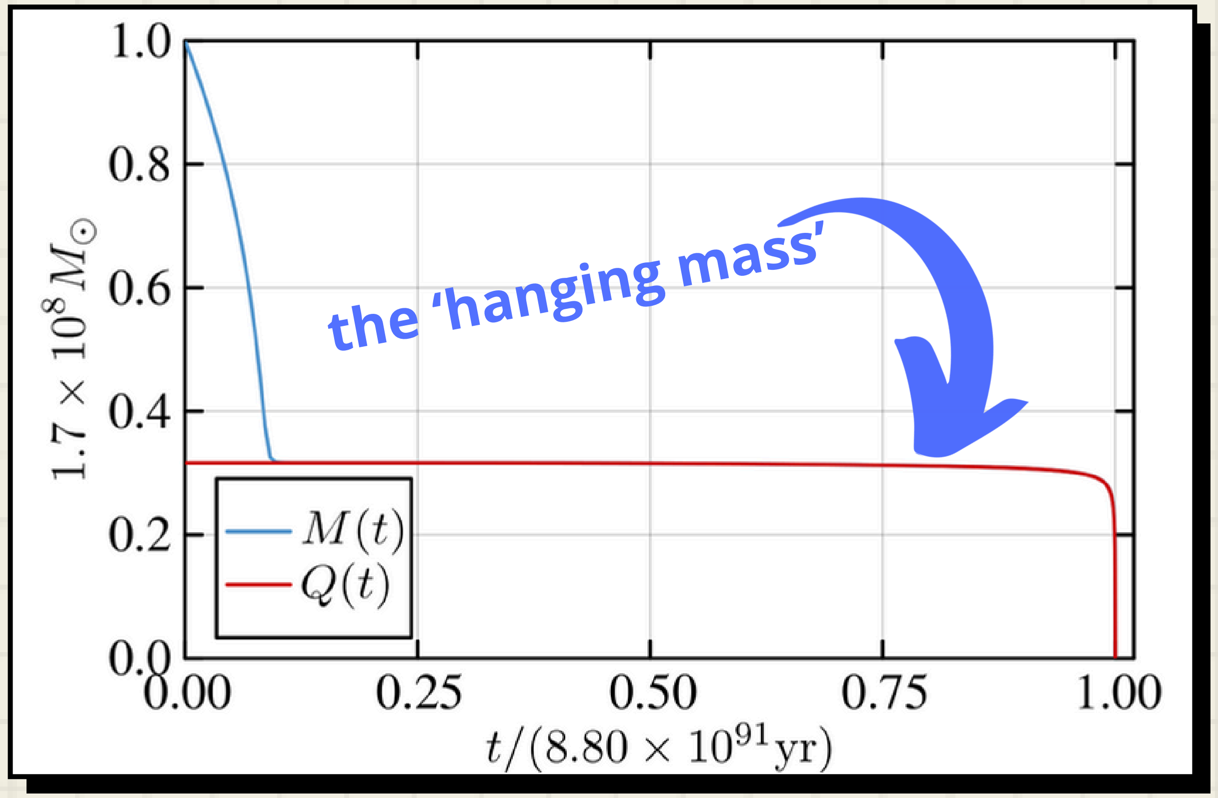
M = $9.0 \times 10^7 M_\odot$, Y = 0.1



M = $1.50 \times 10^8 M_\odot$, Y = 0.1



M = $1.68 \times 10^8 M_\odot$, Y = 0.1

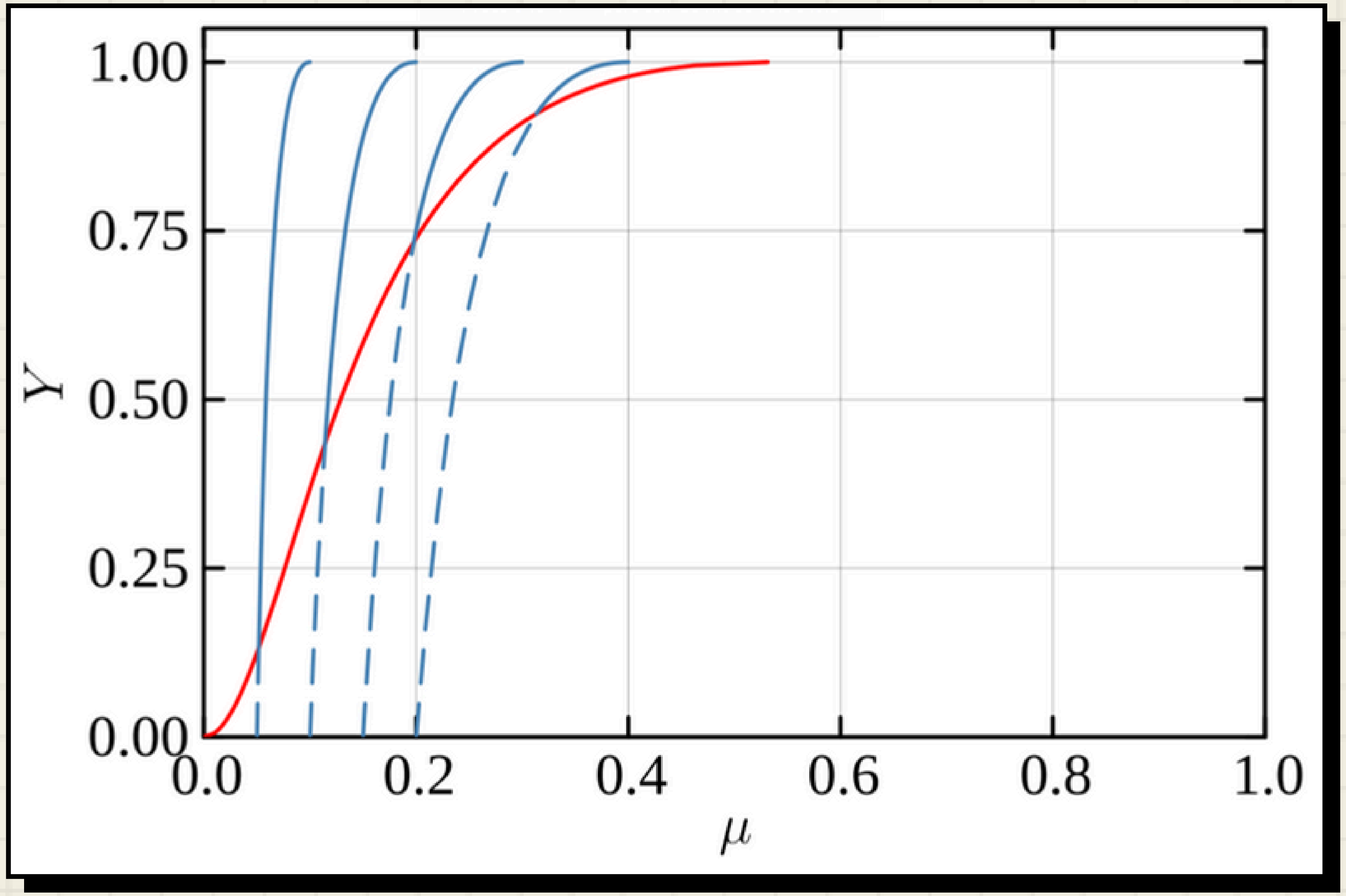


Approximate solutions

$$\frac{\mathrm{d}Y}{\mathrm{d}\mu} = \frac{2S(\mu,Y)\left(\sqrt{1-Y}+1\right)Y^2}{\mu(H(\mu,Y)+S(\mu,Y)Y^2)} - \frac{2Y}{\mu}$$

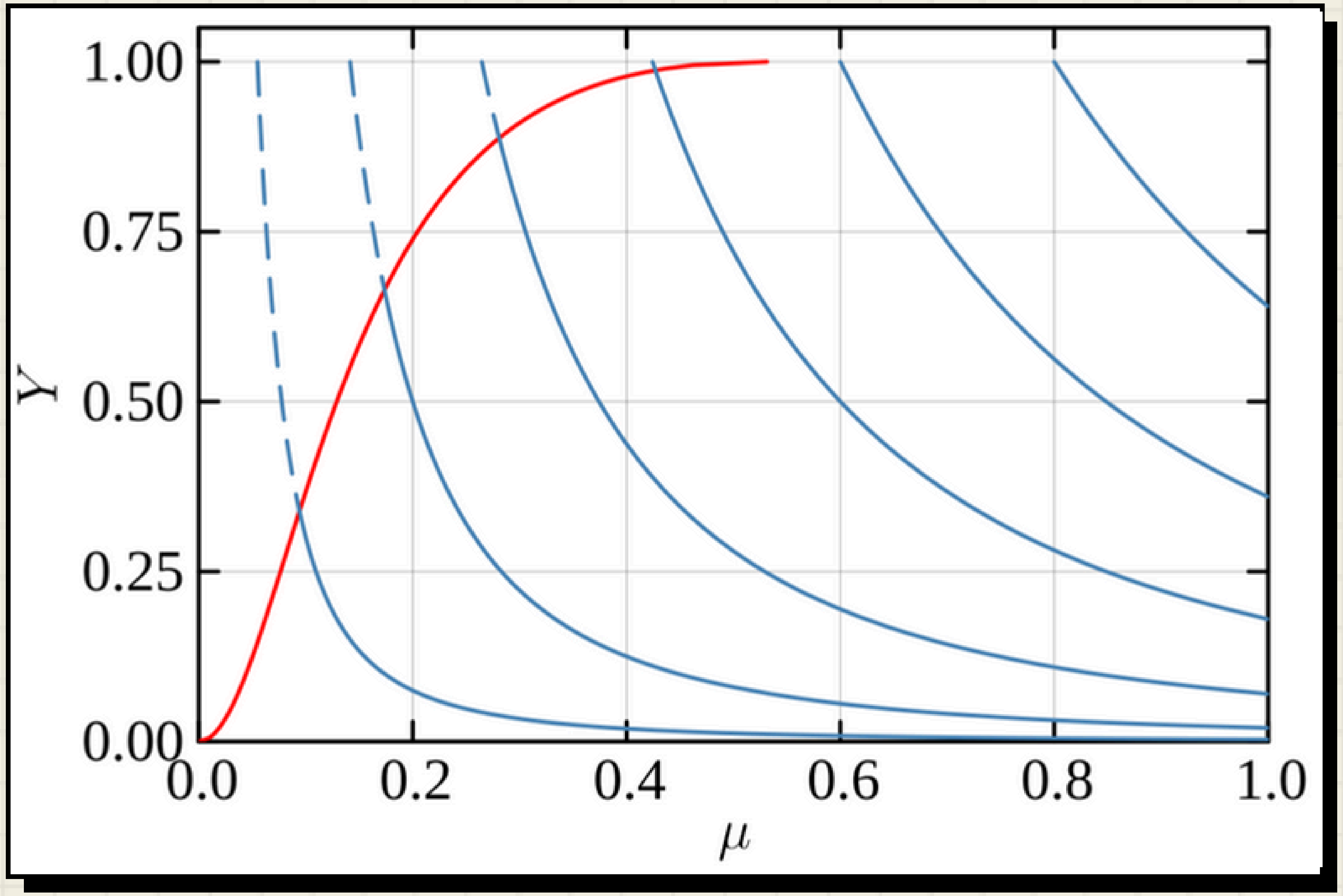
Charge dissipation zone

$$Y_S(\mu) = \frac{(2\mu-\mu_1)\mu_1}{\mu^2}$$



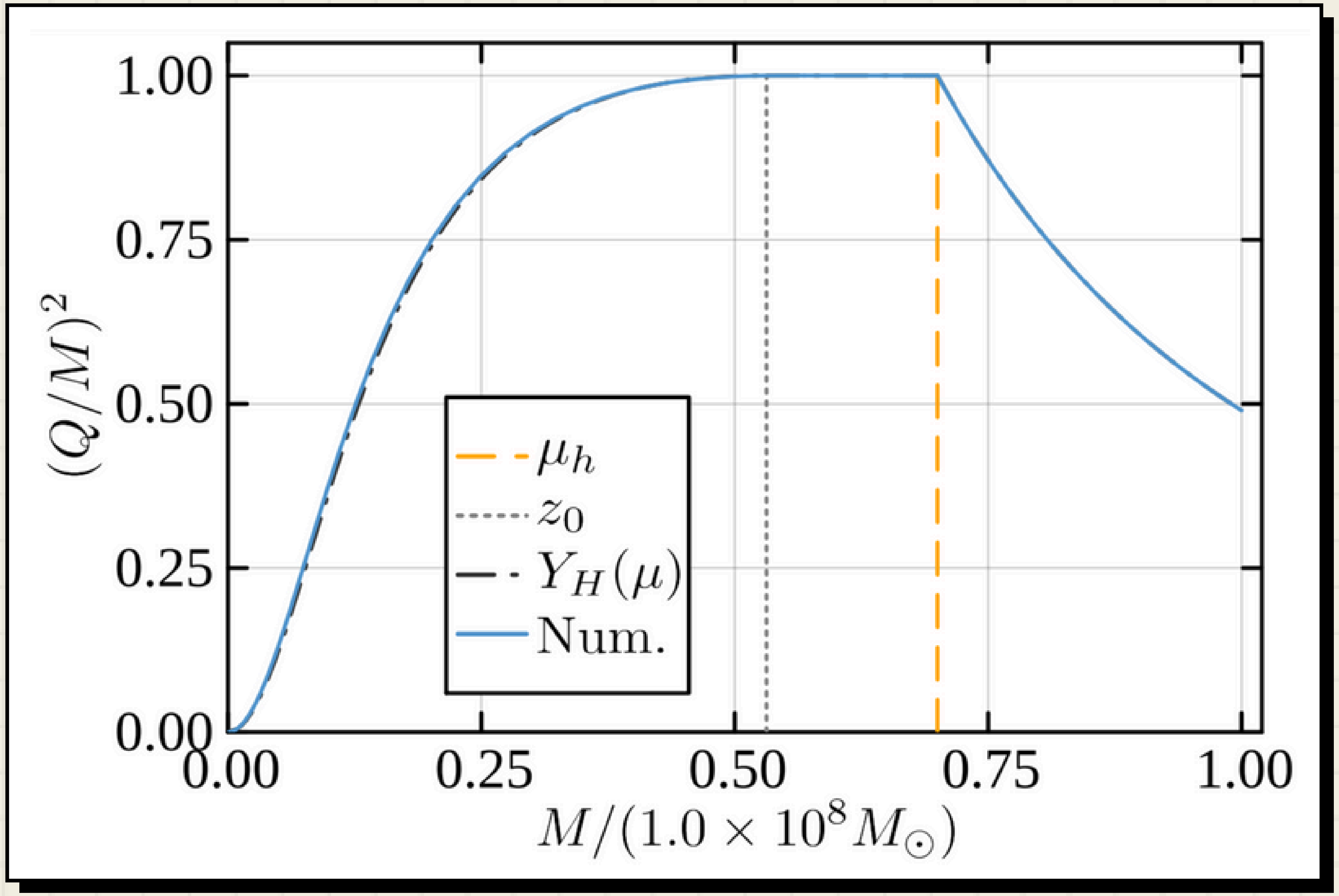
Mass dissipation zone

$$Y_H(\mu) = \left(\frac{\mu_h}{\mu}\right)^2$$



Approximate solutions

$$\frac{\mathrm{d}Y}{\mathrm{d}\mu} = \frac{2S(\mu,Y)\left(\sqrt{1-Y}+1\right)Y^2}{\mu(H(\mu,Y)+S(\mu,Y)Y^2)} - \frac{2Y}{\mu}$$



Approximate attractor curve

$$S(\mu,Y) := \exp\left\{b_0\left[z_0 - \mu\left(\sqrt{1-Y}+1\right)^2/\sqrt{Y}\right]\right\}$$

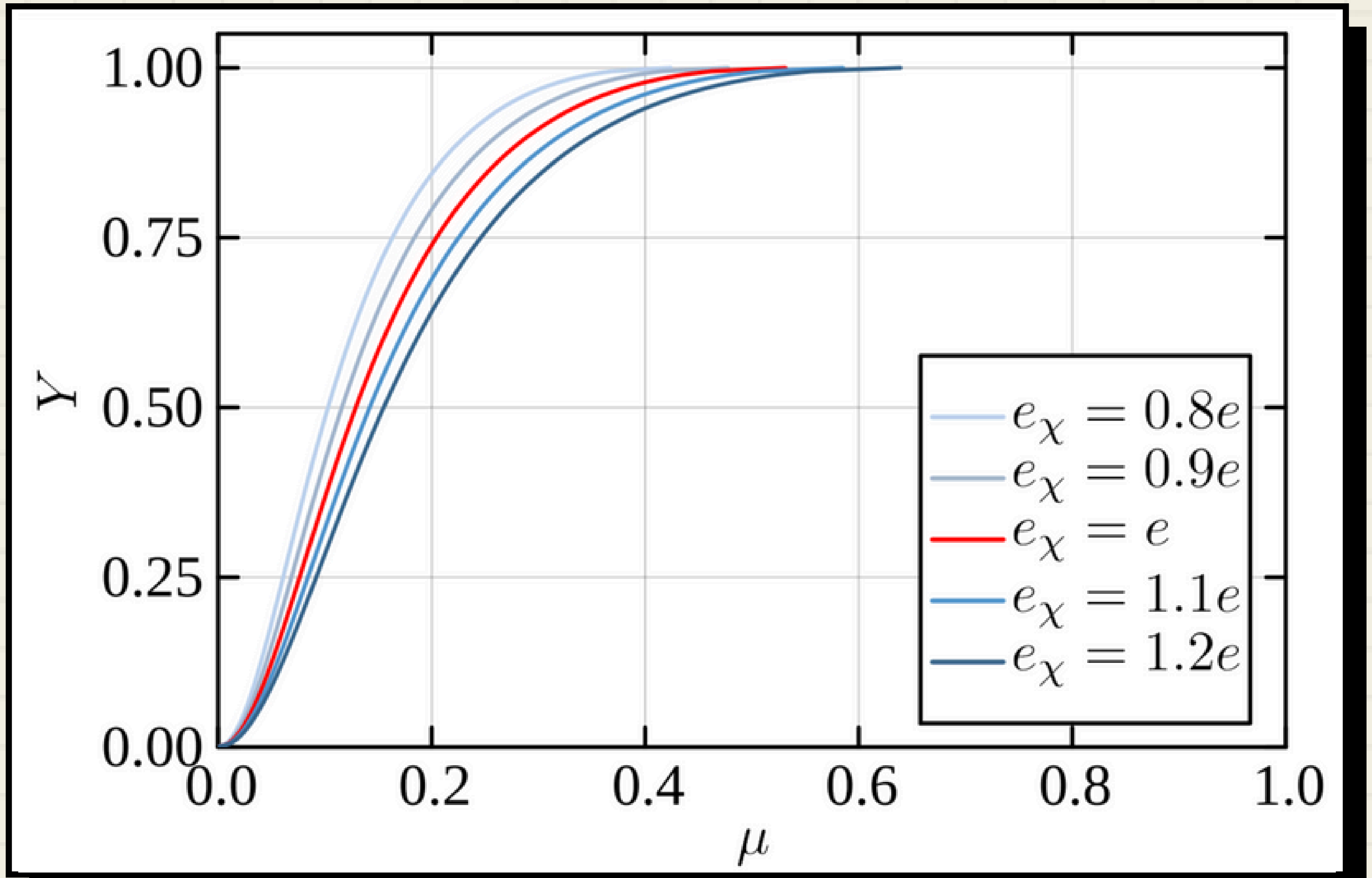
$$z_0 - \frac{\mu\left(\sqrt{1-Y}+1\right)^2}{\sqrt{Y}} \quad \begin{cases} >0 & \Longrightarrow \quad \text{Charge dissipation zone} \\ <0 & \Longrightarrow \quad \text{Mass dissipation zone} \end{cases}$$

$$\mu = z_0 \frac{\sqrt{Y}}{(\sqrt{1-Y}+1)^2}$$

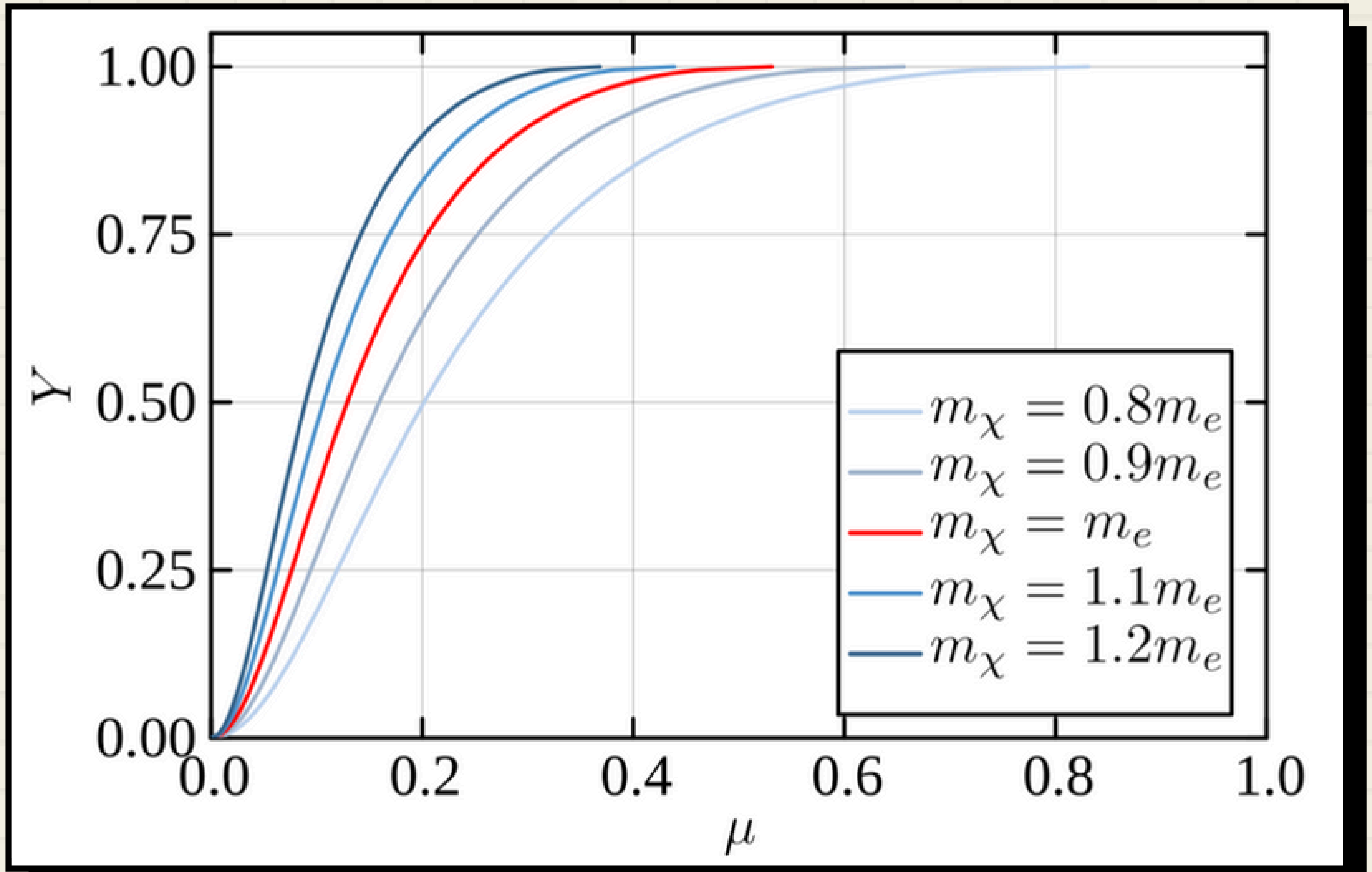
Connection to Dark Matter

$$z_0^{\chi} = \frac{e_\chi \hbar}{\pi m_\chi^2 M_s} \ln\left(\frac{960 e_\chi^4 M_s^2}{\pi^2 \alpha m_\chi^2 \hbar^2}\right) .$$

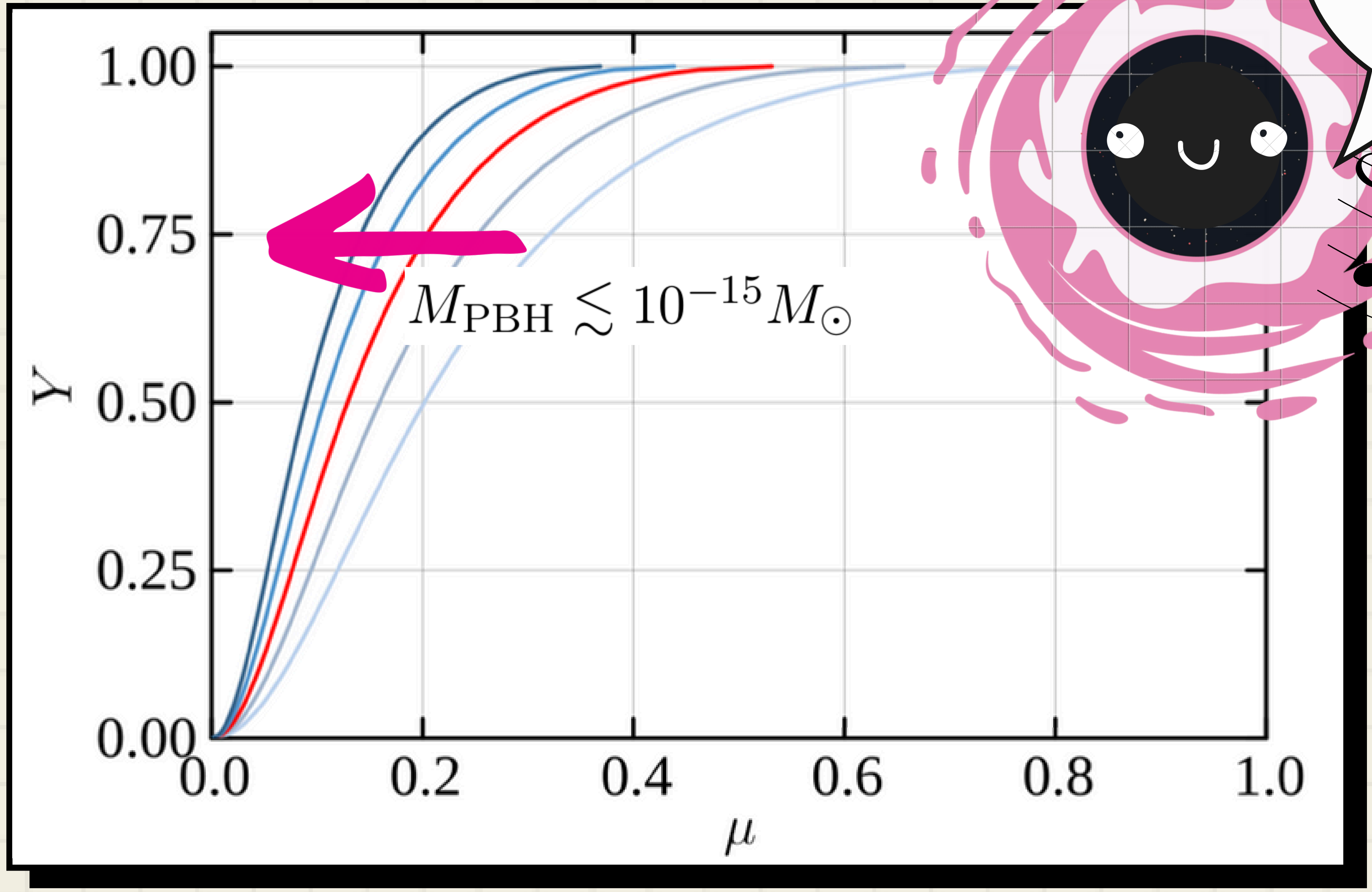
Varying the charge

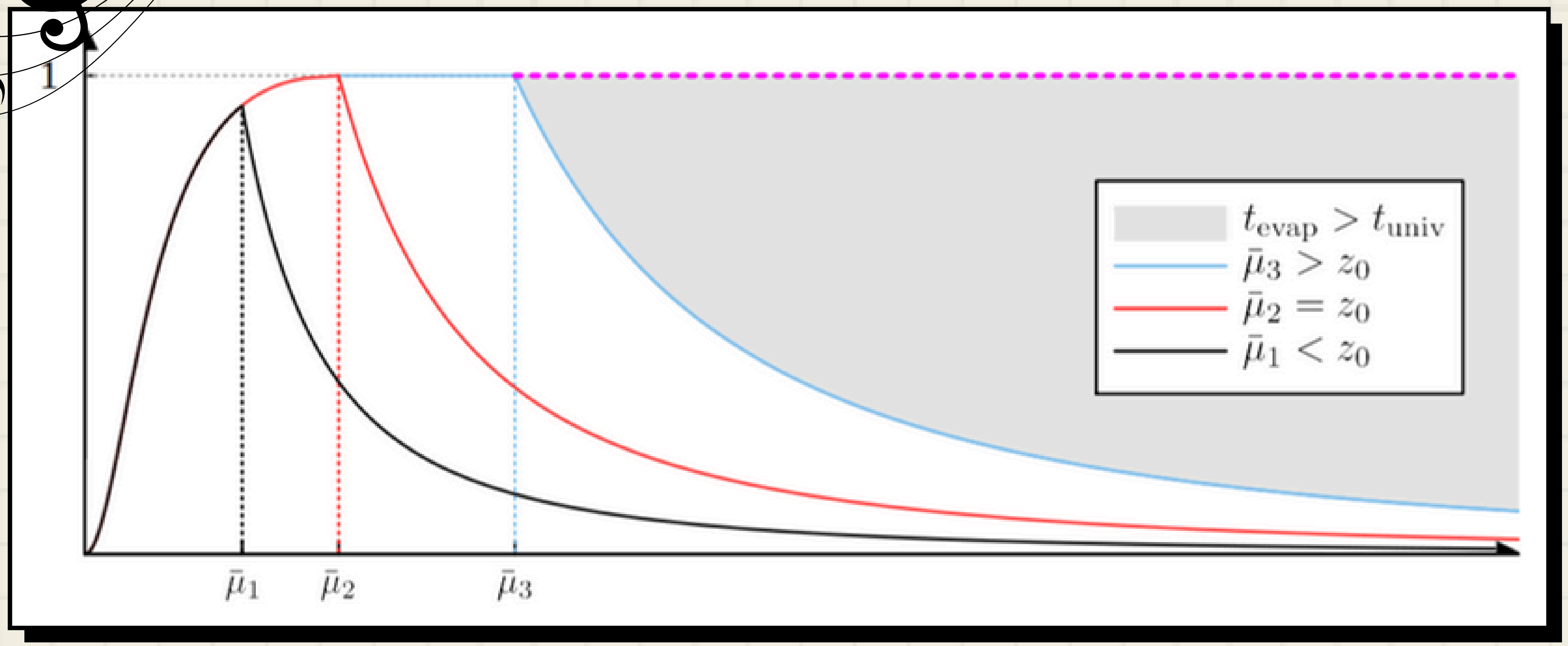


Varying the mass



New PBH mass bounds:





$$\Delta t_{\rm evap}(\bar{\mu}) = \frac{M_s^\chi}{s_0^\chi b_0^\chi} \cdot \begin{cases} \Delta\tau_{\rm schw}(\bar{\mu}) \, , \\ \Delta\tau_{\rm att}(\bar{\mu}) \, , \\ (\exp[(\bar{\mu} - z_0^\chi)\, b_0^\chi] - 1) + \Delta\tau_{\rm att}(z_0) \\ \exp[(\bar{\mu} - z_0^\chi)\, b_0^\chi] \, , \end{cases}$$

$$M_{\rm univ} = \begin{cases} M_{\rm schw} \, , & \bar{\mu} \ll z_0^\chi \\ M_{\rm att}^{\rm univ} , & \bar{\mu} \leq z_0^\chi \\ M_{\rm near\text{-}extr}^{\rm univ} \, , & \bar{\mu} > z_0^\chi \\ M_{\rm extr} \, , & \bar{\mu} - z_0^\chi \gg 1/b_0^\chi \end{cases}$$

Small Y and/or small μ limit

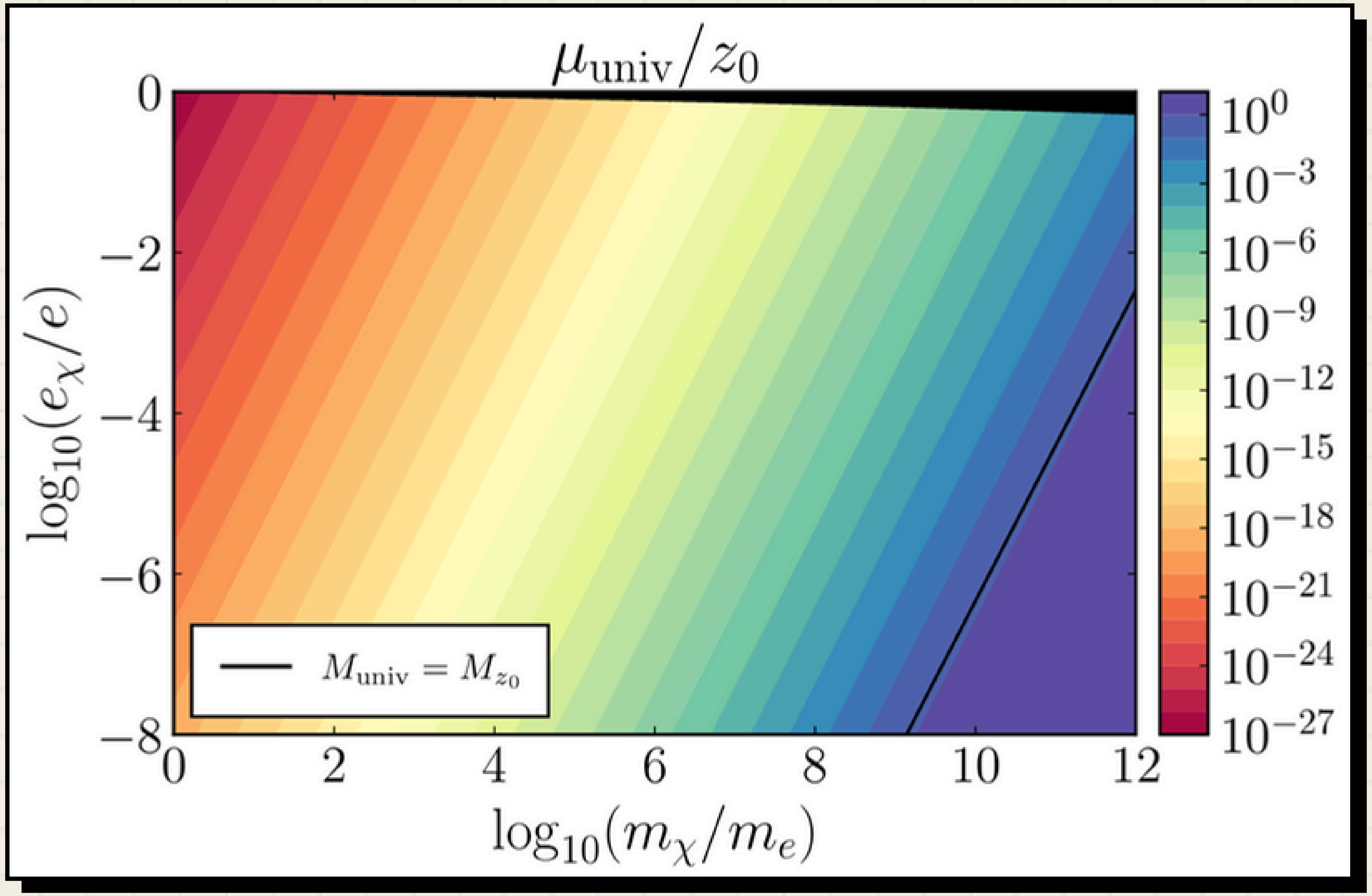
$$\frac{\mathrm{d}\mu}{\mathrm{d}\tau} = -\frac{(H(\mu,Y) + S(\mu,Y)Y^2)}{(\sqrt{1-Y}+1)^4},$$

$$\Delta\tau \overset{Y\to 0}{=} -\int_{\mu}^{0} \frac{32}{27}\tilde{\mu}^2\,\mathrm{d}\tilde{\mu}$$

$$\to \quad \Delta\tau_{\mathrm{schw}} \overset{Y\to 0}{=} \frac{32}{81}\mu^3$$

$$\Delta t = \frac{M_s^\chi}{s_0^\chi}\,\Delta\tau_{\mathrm{schw}} = \frac{1920\pi}{\alpha\hbar}\left[\frac{32}{81}(M_s^\chi\mu)^3\right]$$

$$= \frac{1920\pi}{\alpha\hbar}\left[\frac{32}{81}M^3\right] = \Delta t_{\mathrm{schw}}$$



Approximate attractor curve

$$\mu = z_0 \frac{\sqrt{Y}}{(\sqrt{1-Y}+1)^2}$$

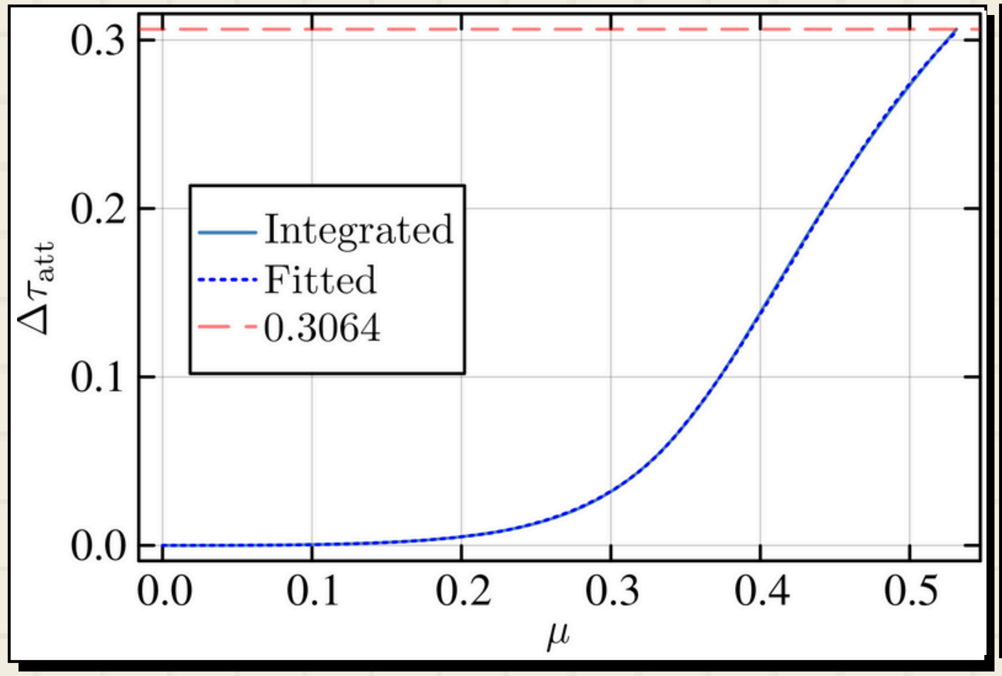
$$S(\mu, Y) \sim 1$$

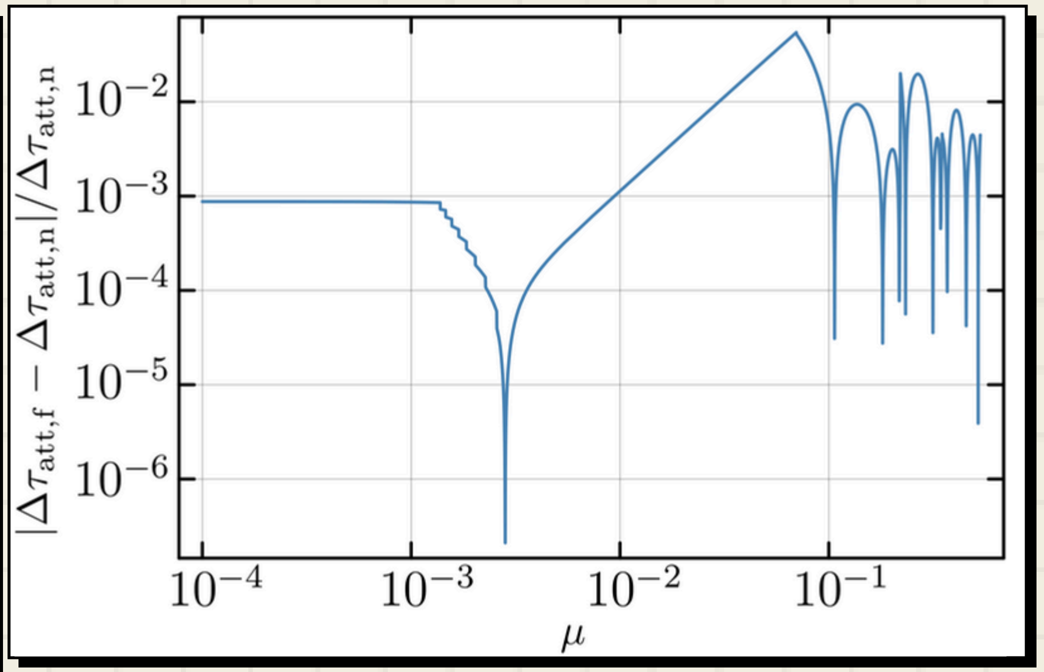
$$\mathrm{d}\tau = -\frac{(\sqrt{1-Y}+1)^4\,\mathrm{d}\mu}{(H(\mu,Y)+Y^2)}$$

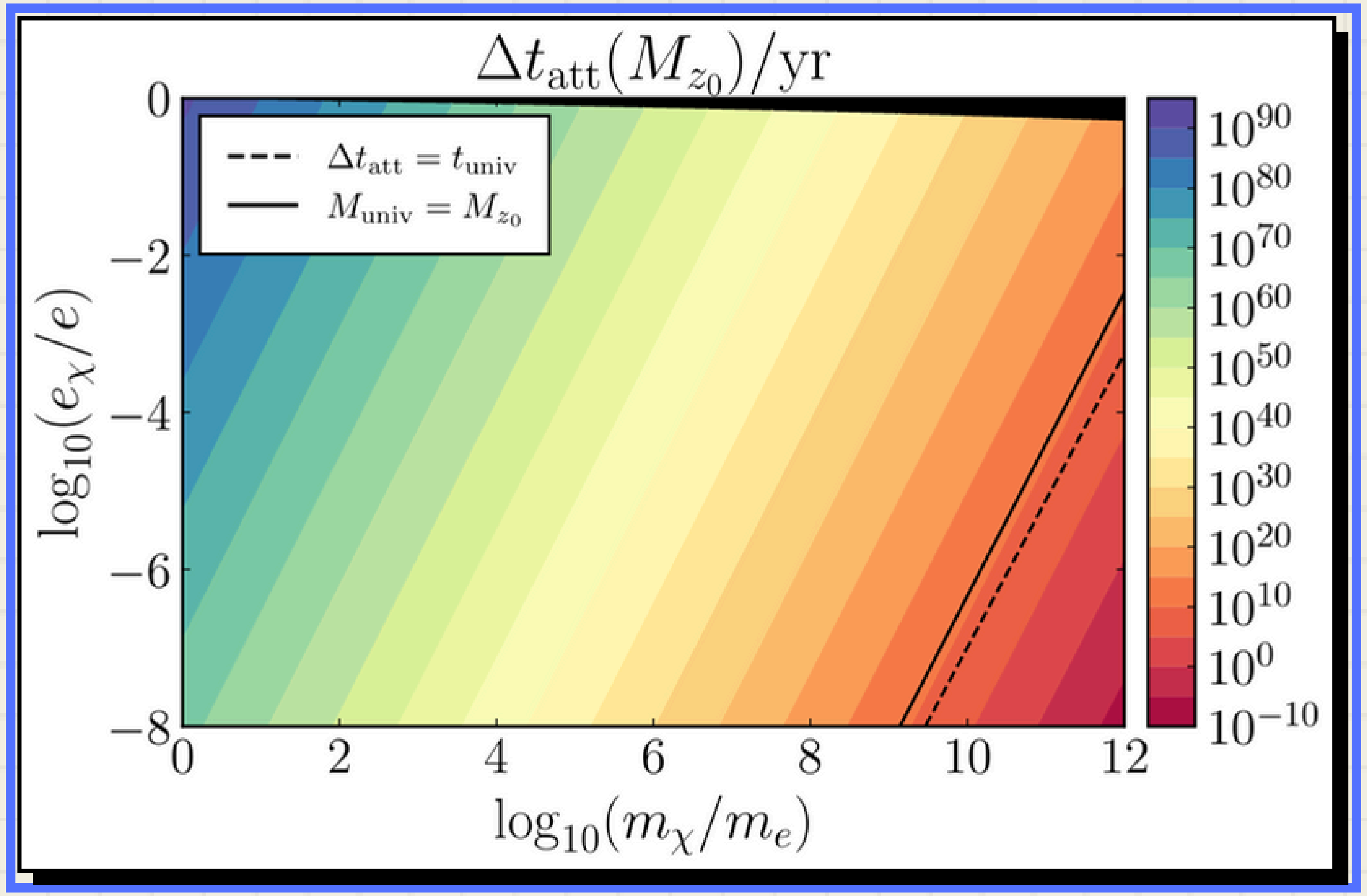
$$\Delta\tau_{\mathrm{att}}(\mu) = \frac{1}{a + b\,\mu^{-m}}$$

Approximate time estimate

$\mu < 0.05$	->	(a, b, m) = (0,81/32,3)
$0.05 < \mu < 0.38$	->	(a, b, m) = (-1.54,0.1045,4.765)
$\mu > 0.38$	->	(a, b, m) = (2.308,0.0277,5.54)





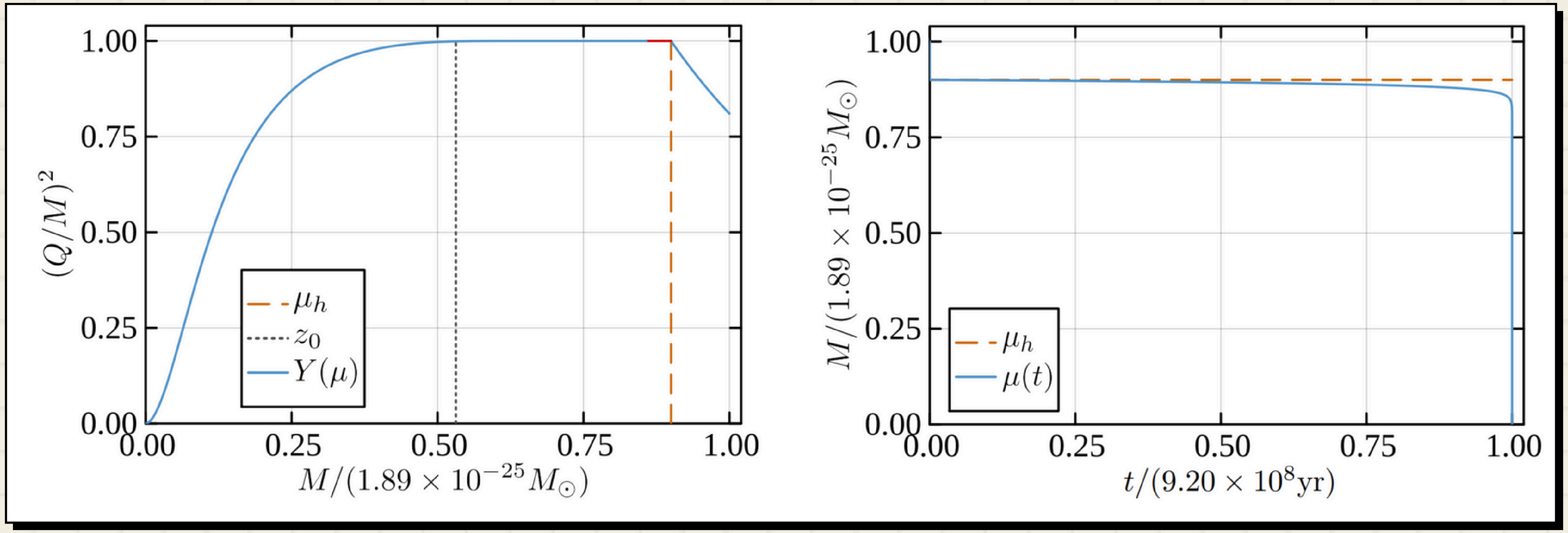


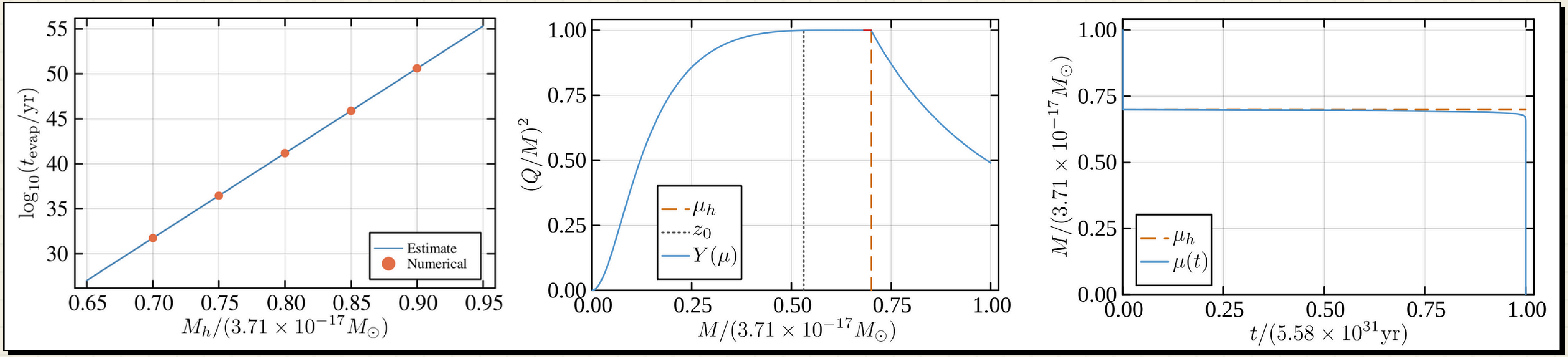
Near extremal evolution

Near-extremal evolution:

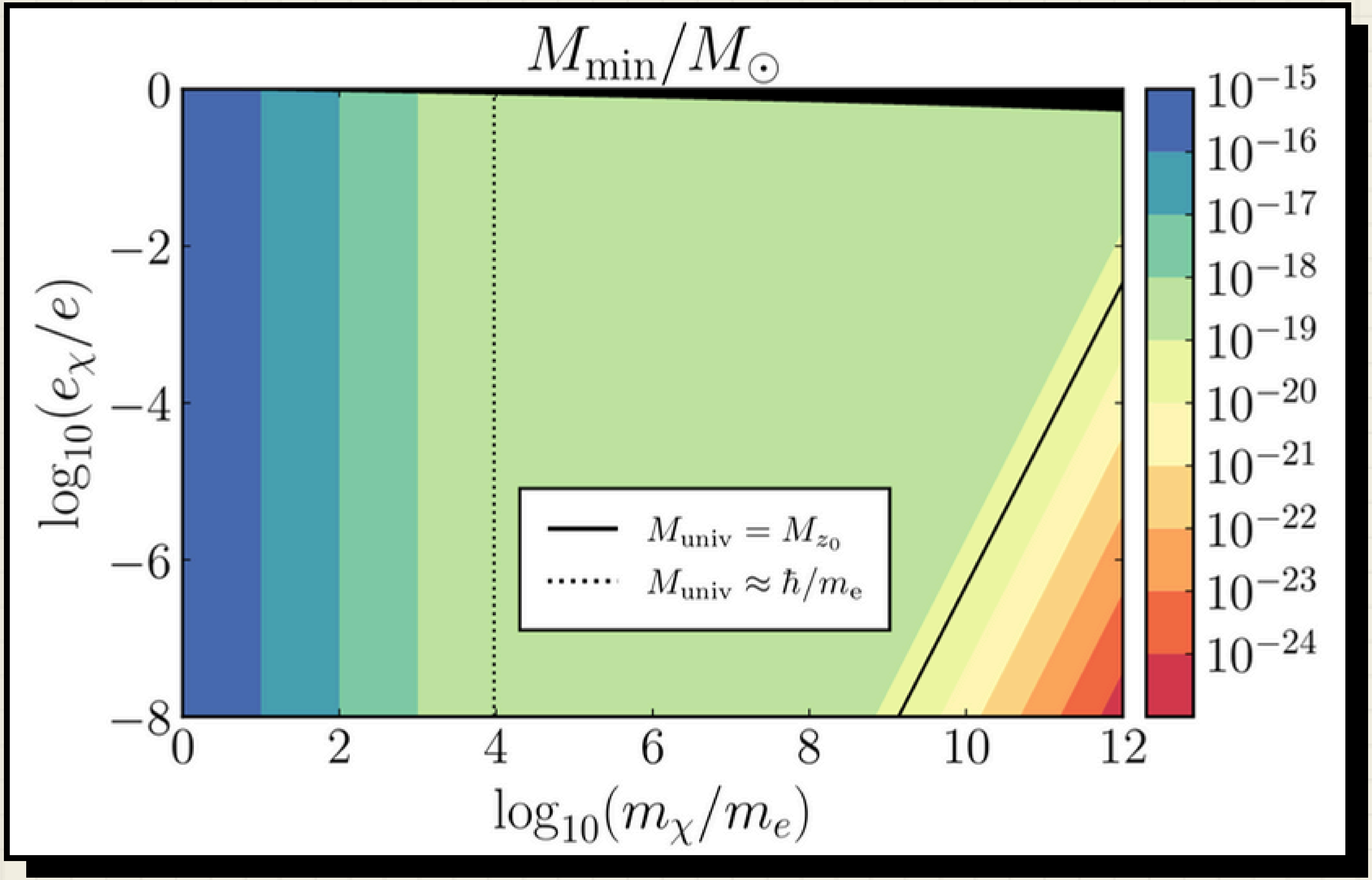
$$\frac{\mathrm{d}\mu}{\mathrm{d}\tau} = -\exp[(z_0 - \mu)b_0]$$

$$\Delta t \sim \frac{M_s}{s_0^\chi \, b_0^\chi} \exp[(\mu_\mathrm{h} - z_0^\chi)\, b_0^\chi]$$





Long Live (tiny) Black Holes!



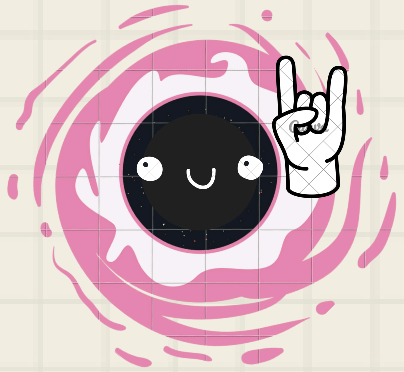
Conclusions

- Reissner-Nordstrom black holes have very interesting evaporation evolution for certain mass ranges;

- If PBHs can form with an initial dark U(1) charge, they would not discharge via accreation effects, allowing them to follow the mass dissipation curve and achieve near-extremality;

- Given the right combination of dark-electron charge, mass and initial PBH mass, PBHs of masses below 10^(-15) solar masses can still account for the total amount of dark matter in the universe.

- And Long Live Black Holes!



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The Schwinger effect validity region

Electric field gradients must be small compared to the Compton wavelength of the lightest electrically charged particle

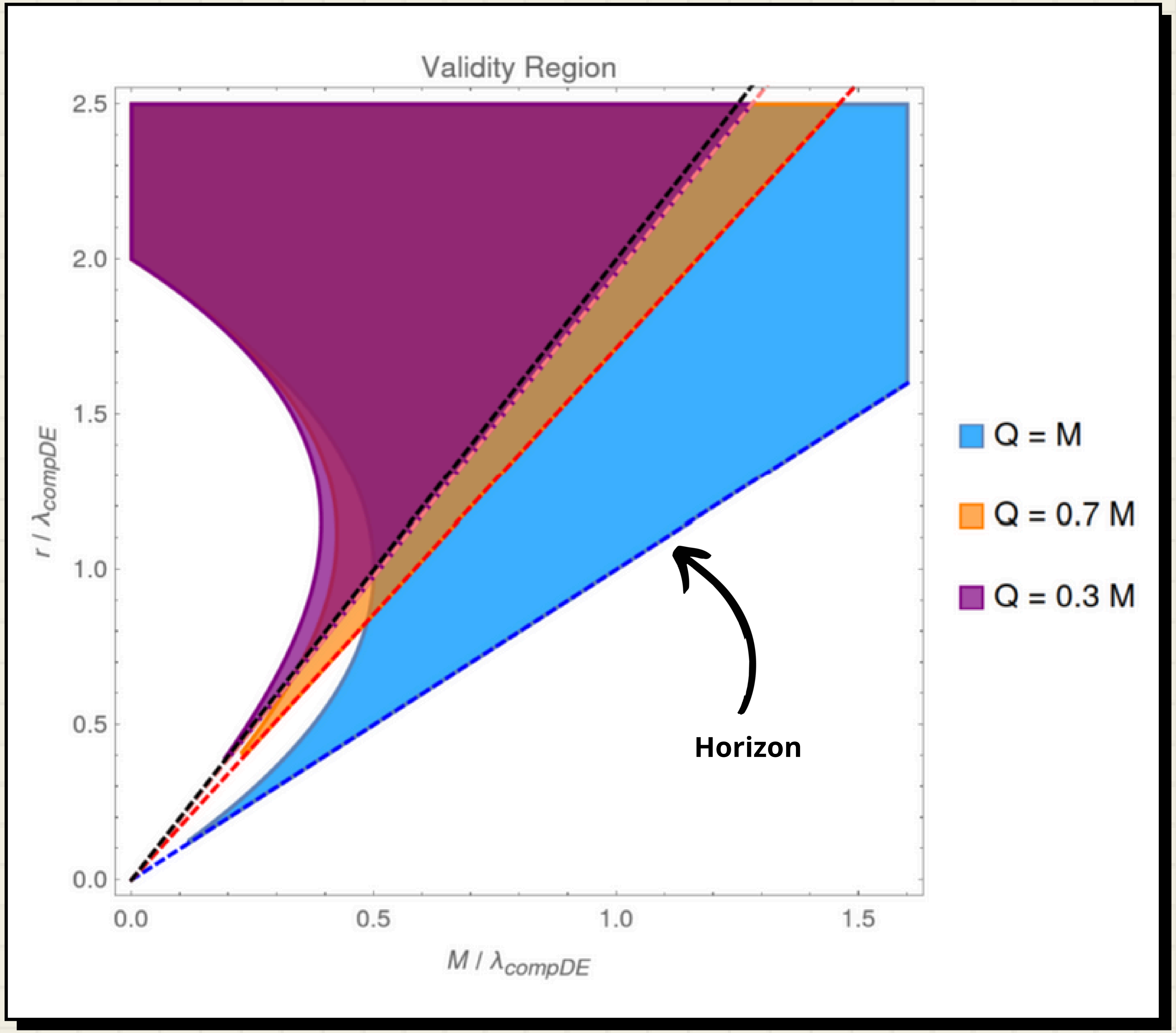
$$\frac{1}{E}\frac{\mathrm{d}E}{\mathrm{d(proper\ length)}} < \frac{1}{\lambda_{\mathrm{compDE}}} = \frac{m_\chi}{\hbar c}.$$

$$\sqrt{1-\frac{2M}{r}+\frac{Q^2}{r^2}}\,\frac{2}{r} < \frac{1}{\lambda_{\mathrm{compDE}}} = \frac{m_\chi}{m^2_{\mathrm{Planck}}}$$

$$r^4 - 4\lambda^2_{\mathrm{compDE}}(r^2 - 2Mr + Q^2) > 0$$

r > r+

$$r^2 - 2Mr + Q^2 > 0$$



Rescaling relations

Varying the parameters

$$M_s^\chi = \sigma_M M_s$$
$$m_\chi = \sigma_m m_e$$
$$e_\chi = \sigma_e e$$

$$z_0^\chi = \frac{e_\chi \hbar}{\pi m_\chi^2 M_s} \ln\left(\frac{960 e_\chi^4 M_s^2}{\pi^2 \alpha m_\chi^2 \hbar^2}\right) \qquad M_{z_0} = z_0 M_s$$

$$s_0^\chi = \frac{s_0}{\sigma_M^2}, \qquad z_0^\chi = \frac{\sigma_e}{\sigma_m^2 \sigma_M}\left[z_0 + \frac{2}{b_0}\ln\left(\frac{\sigma_e^2 \sigma_M}{\sigma_m}\right)\right], \qquad b_0^\chi = \frac{\sigma_m^2 \sigma_M}{\sigma_e} b_0$$

$$\xi = \sigma_e / \sigma_m$$

$$\sigma_e = \frac{\vartheta}{\sigma_M \xi}, \qquad \sigma_m = \frac{\vartheta}{\sigma_M \xi^2}, \qquad \xi = \xi(\vartheta) = \left(\frac{b_0 z_0 \vartheta}{b_0 z_0 + 2\ln\vartheta}\right)^{1/3}$$

$$z_0^\chi = z_0 \qquad M_{z_0}^\chi = \frac{\vartheta^{1/3}}{\sigma_m}\left(\frac{b_0 z_0 + 2\ln\vartheta}{b_0 z_0}\right)^{2/3} M_{z_0}$$

Regarding the discreteness of states near extremality

Breaking down energy

$$M - Q \lesssim E_{\text{brk.}} \equiv \frac{M_{Pl}}{Q^3}.$$

(from 2411.03447 in weird units)

$$M - Q \gg m_{\text{Planck}}^4 / Q^3$$

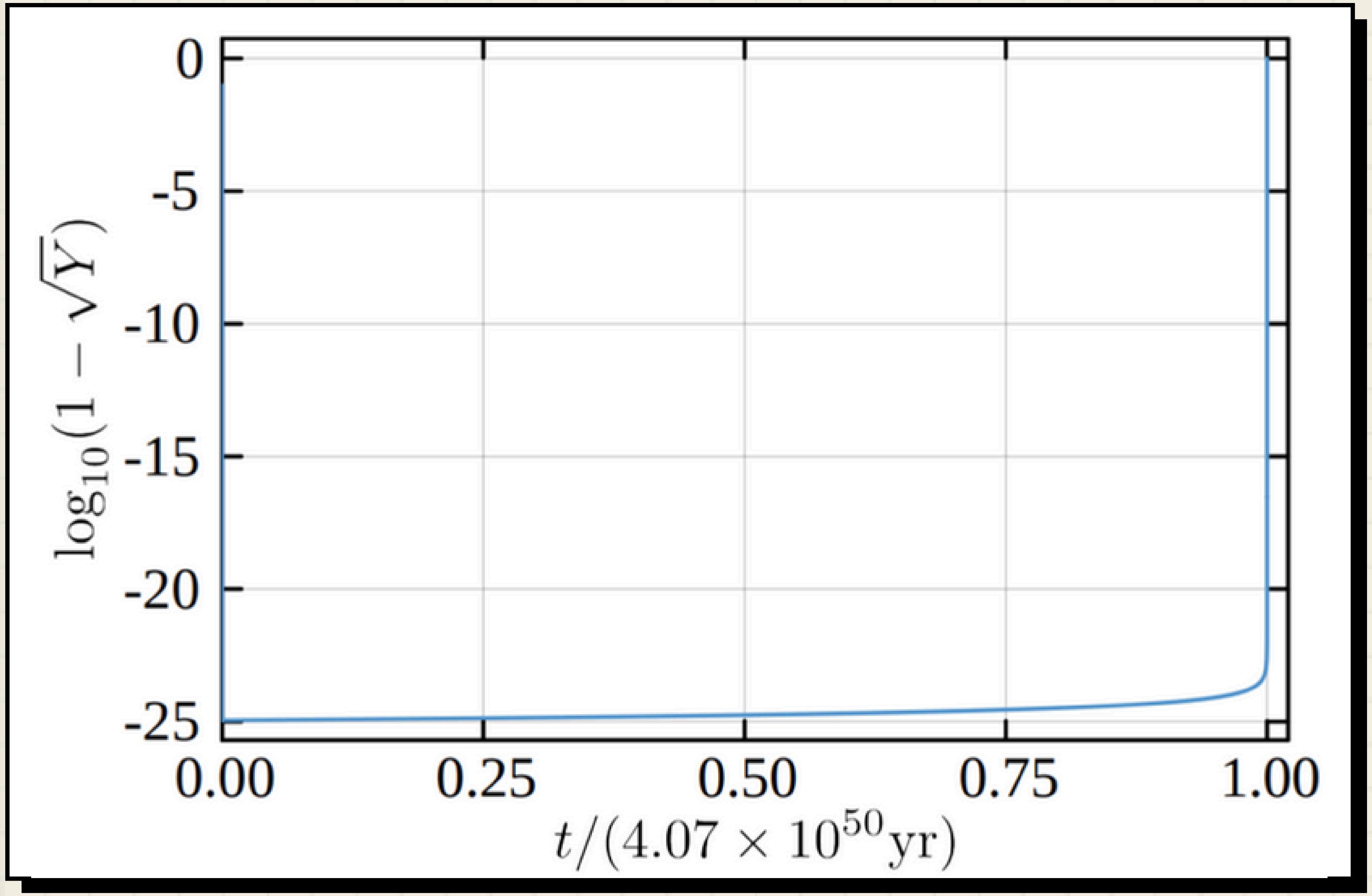
$$Y^{3/2}(1 - \sqrt{Y}) \gg (m_{\text{Planck}}/M)^4$$

In our analysis

smallest Y value $\sqrt{Y} \sim 1 - 10^{-25}$

smallest mass $10^{-25} M_{\odot} \sim 10^{13} m_{\text{Planck}}$

$$(m_{\text{Planck}}/M)^4 \sim 10^{-52}$$

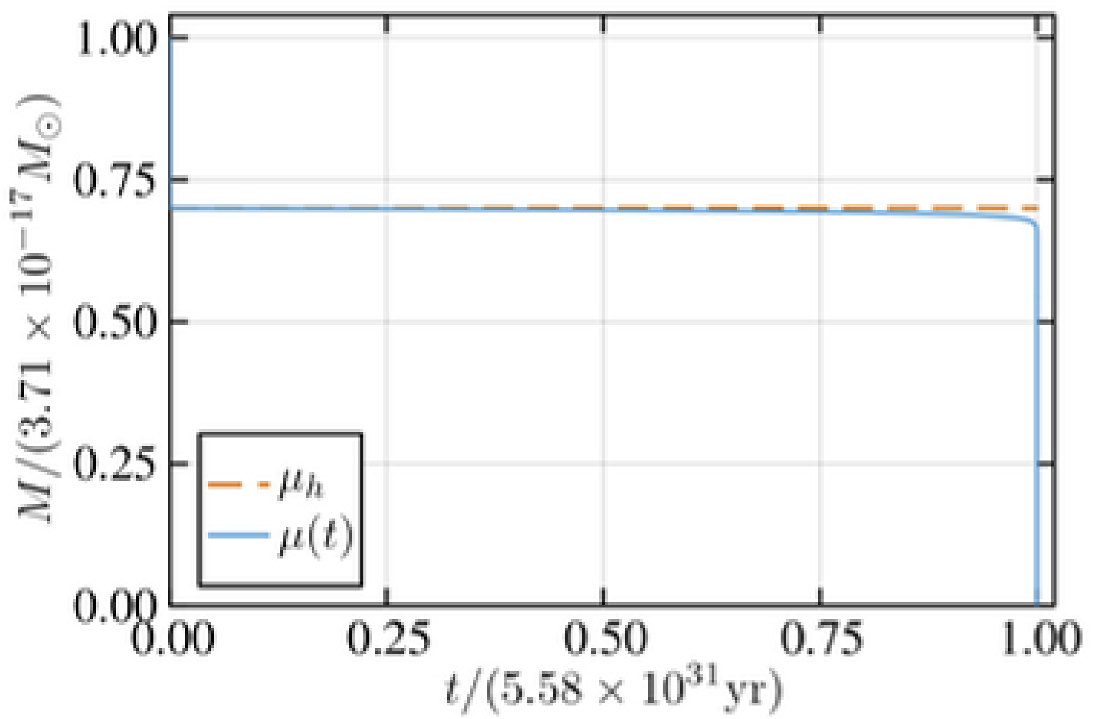


Near extremal evolution

Evaporation times

ϱ/ϱ_0	$\mu_h = 0.70$	$\mu_h = 0.75$	$\mu_h = 0.80$	$\mu_h = 0.85$	$\mu_h = 0.90$
10^{-14}	5.58×10^{31}	2.90×10^{36}	1.51×10^{41}	7.83×10^{45}	4.07×10^{50}
10^{-15}	4.94×10^{23}	6.82×10^{27}	9.42×10^{31}	1.30×10^{36}	1.80×10^{40}
10^{-16}	4.16×10^{15}	1.51×10^{19}	5.54×10^{22}	2.03×10^{26}	7.43×10^{29}
10^{-17}	3.29×10^{7}	3.12×10^{10}	3.00×10^{13}	2.90×10^{16}	2.80×10^{19}
10^{-18}	2.43×10^{-1}	5.80×10^{1}	1.45×10^{4}	3.64×10^{6}	9.20×10^{8}

Table 1: Evaporation times (in years) as a function of the 'hanging mass' μ_h and charge to mass ratio, where $\varrho = e_\chi/m_\chi$ is the rescaled charge to mass ratio, and $\varrho_0 = 2.04 \times 10^{21}\sqrt{4\pi\varepsilon_0 G}$ corresponds to the standard value in electrodynamics. In each case, the electron charge is rescaled by a factor $\sigma_e = 10^{-4}$.



Evaporation times: approximation vs. numerical results

ϱ/ϱ_0	$\mu_h = 0.70$	$\mu_h = 0.75$	$\mu_h = 0.80$	$\mu_h = 0.85$	$\mu_h = 0.90$
10^{-14}	4.83×10^{-6}	1.26×10^{-7}	3.34×10^{-9}	8.99×10^{-11}	3.25×10^{-12}
10^{-15}	2.52×10^{-5}	1.00×10^{-6}	4.05×10^{-8}	1.65×10^{-9}	6.90×10^{-11}
10^{-16}	1.46×10^{-4}	8.67×10^{-6}	5.28×10^{-7}	3.28×10^{-8}	2.06×10^{-9}
10^{-17}	1.13×10^{-3}	9.25×10^{-5}	8.13×10^{-6}	7.45×10^{-7}	7.03×10^{-8}
10^{-18}	4.98×10^{-2}	3.12×10^{-3}	2.33×10^{-4}	2.59×10^{-5}	3.31×10^{-6}