BREAKING THE "DISTANCE DUALITY RELATION" TO EXPLAIN COSMIC TENSIONS

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Based on:

- [arxiv:2504.10464] with: William Giarè, Natalie Hogg, Thomas Montandon, Adèle Poudou and Vivian Poulin
- [arxiv:2505.02909] with: Ruchika, William Giarè, and Alessandro Melchiorri

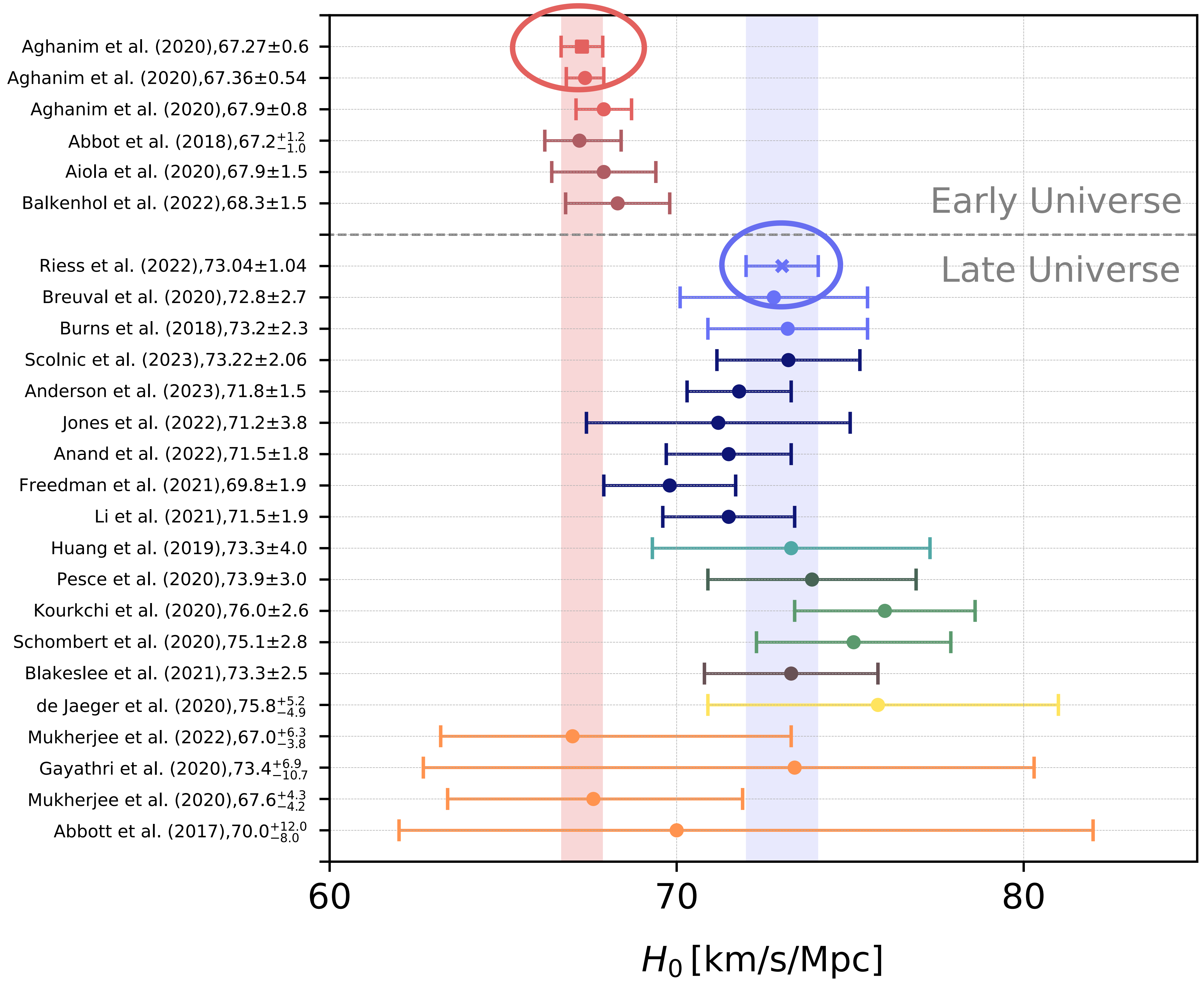
Illustrations: Inês Viegas Oliveira (ivoliveira.com)

The Hubble Tension

The “Hubble Tension”

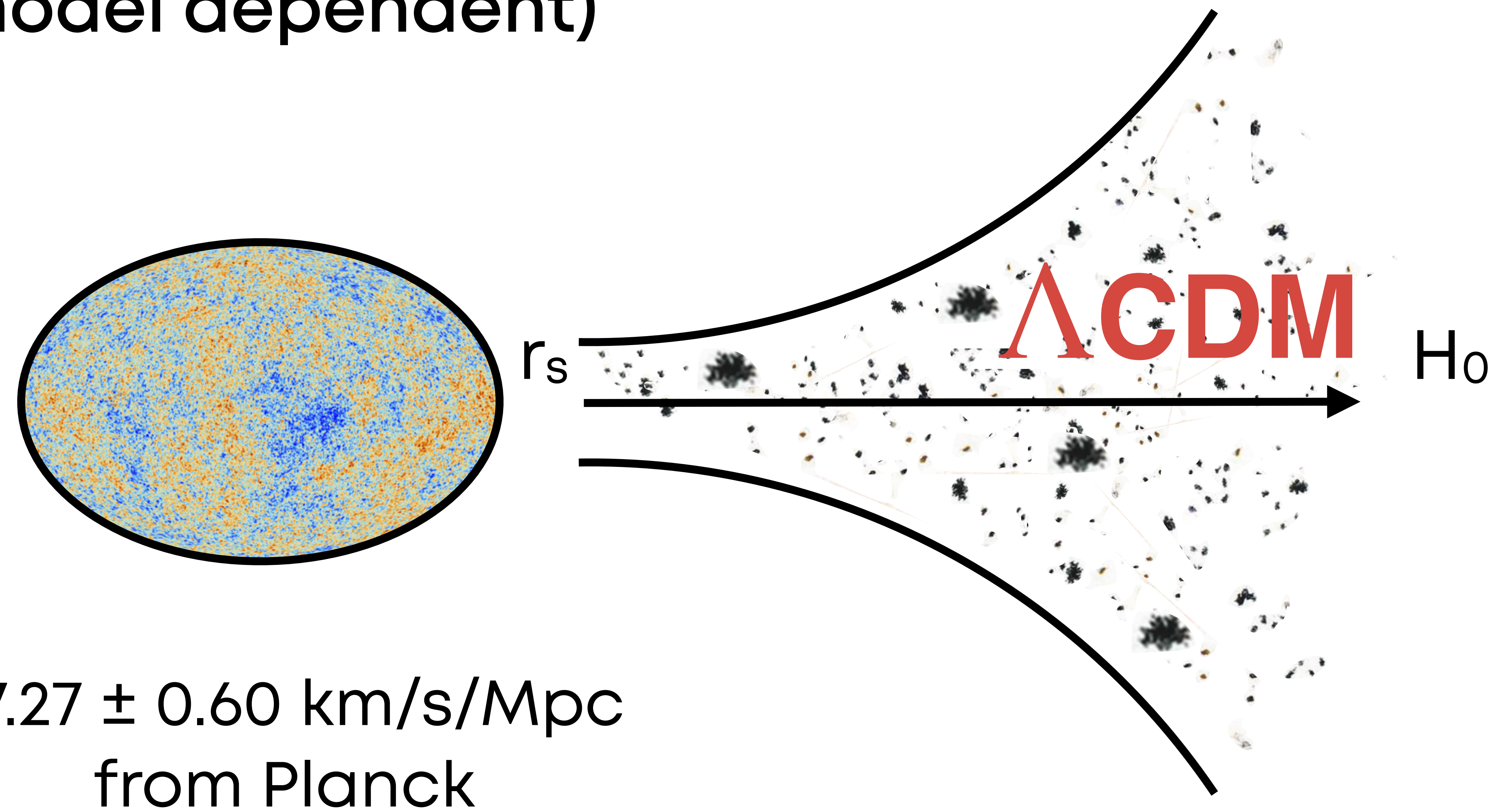
Unreconcilable values for H_0 from the CMB and from direct local distance ladder measurements

- ~5σ tension between Planck 2018 and SH$_0$ES:
  - CMB (Planck): $H_0 = 67.27 \pm 0.60$ km/s/Mpc
  - SNe (R22): $H_0 = 73.04 \pm 1.04$ km/s/Mpc
- The CMB data assumes the ΛCDM model
- DESI BAO (+BBN+CMB): $H_0 = 68.45 \pm 0.47$ km/s/Mpc [DESI Collaboration DR2 2025: arXiv:2503.14738]
- Compilation of early vs late time data that disagree
- But how do we measure H_0 in each case?

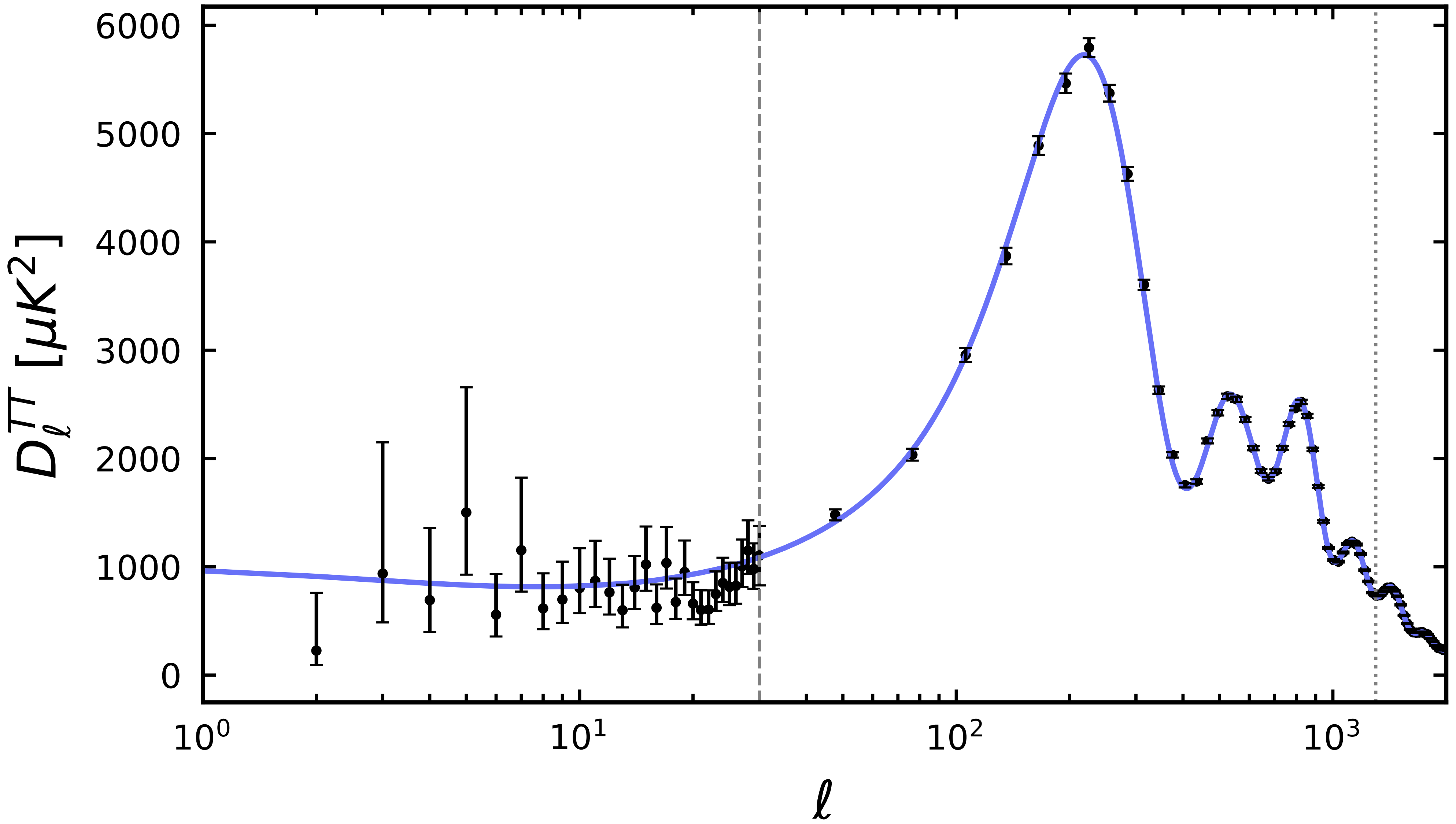


The "Hubble Tension"

CMB
(model dependent)



67.27 ± 0.60 km/s/Mpc
from Planck

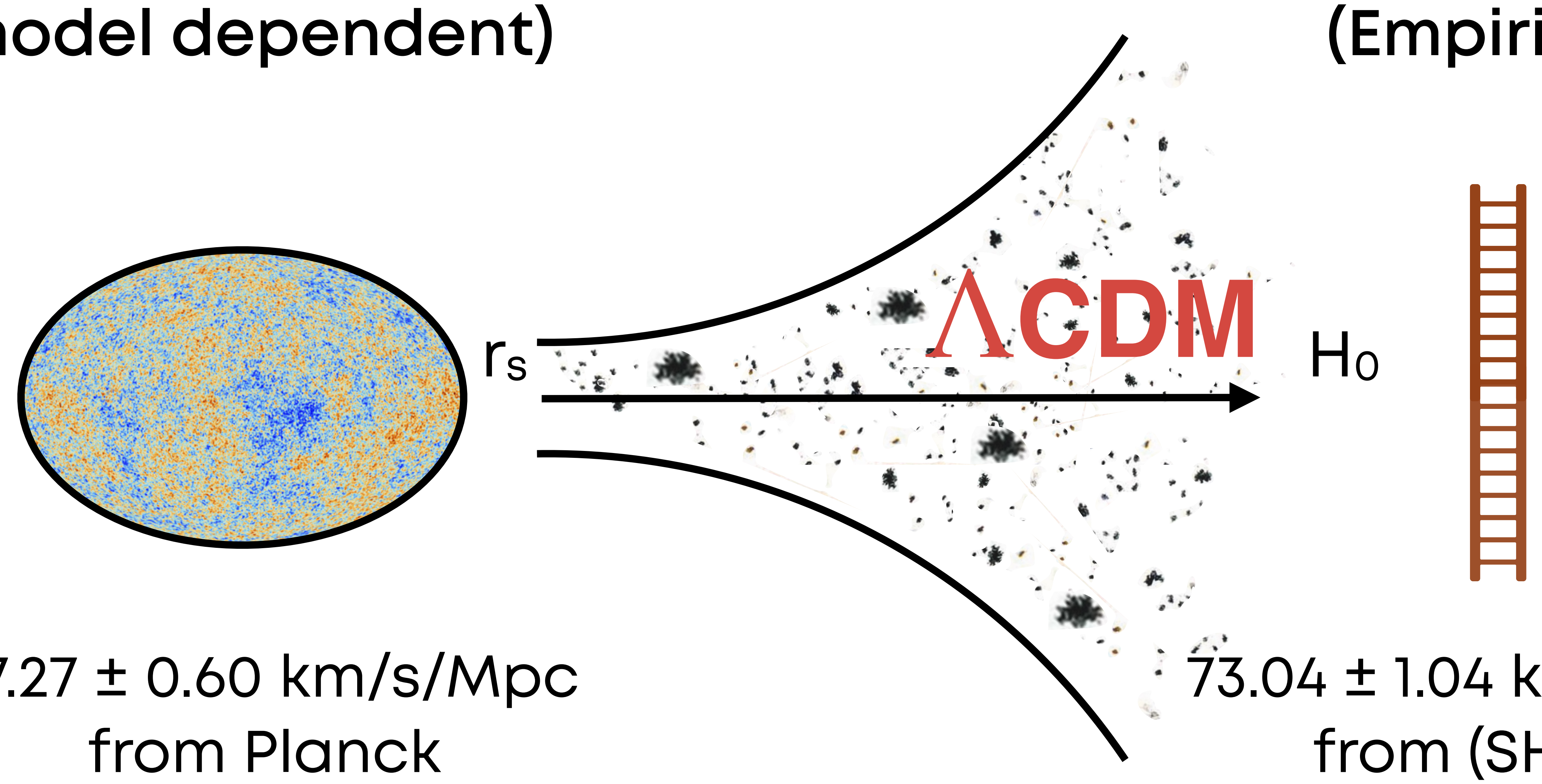


[Aghanim et al.: Astron.Astrophys. 641 (2020) A6]

The “Hubble Tension”

CMB
(model dependent)

Local
(Empirical)



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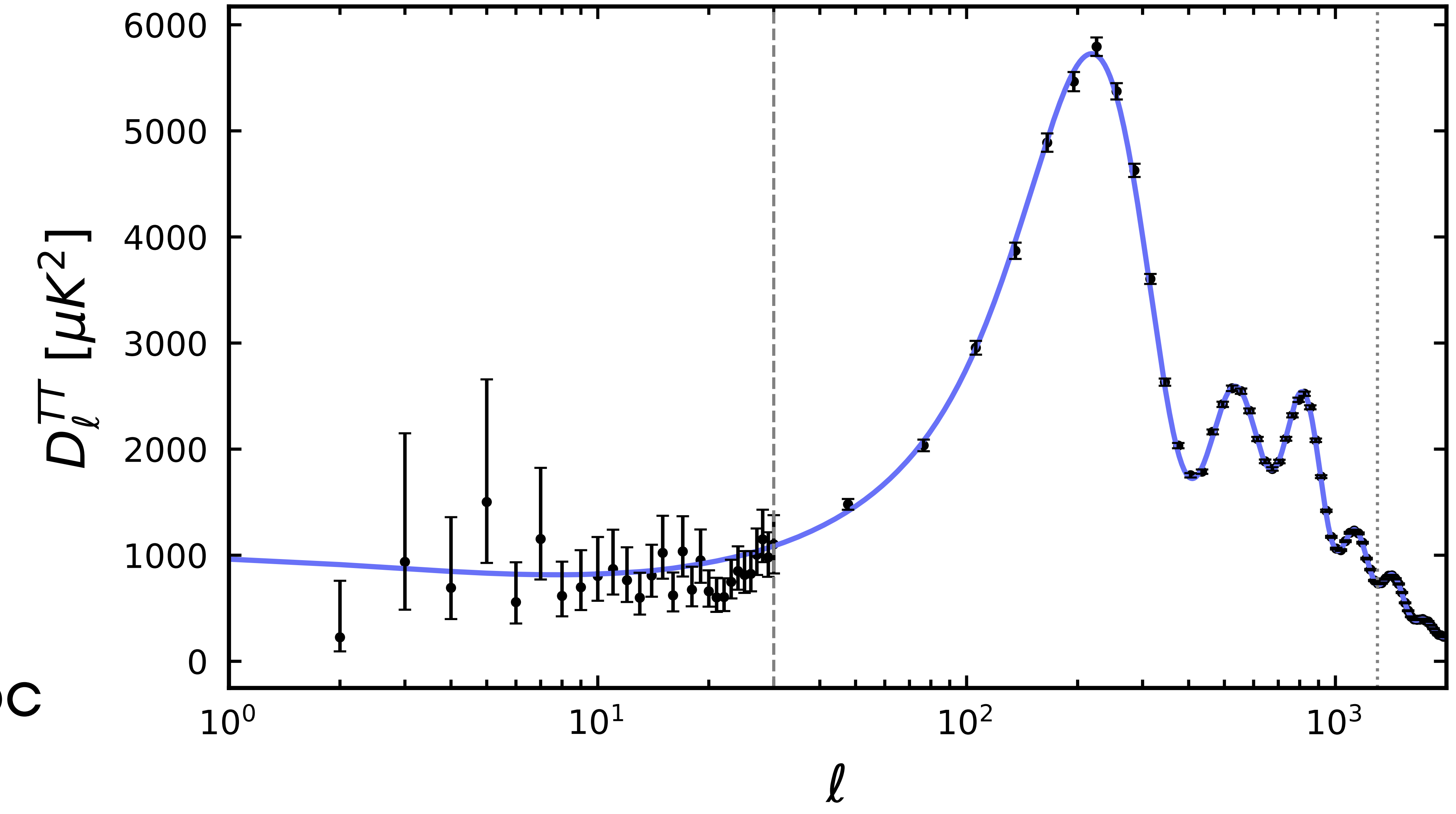
73.04 ± 1.04 km/s/Mpc
from (SH0ES)



- Infer H_0 from the cosmological distance ladder
- Based on local distance measurements and astrophysical observables/calibrations

[Aghanim et al.: Astron.Astrophys. 641 (2020) A6]

Elsa Teixeira • LUPM (CNRS / U. Montpellier)
CosmoVerse@Istanbul • İTÜ • 26/06/2025

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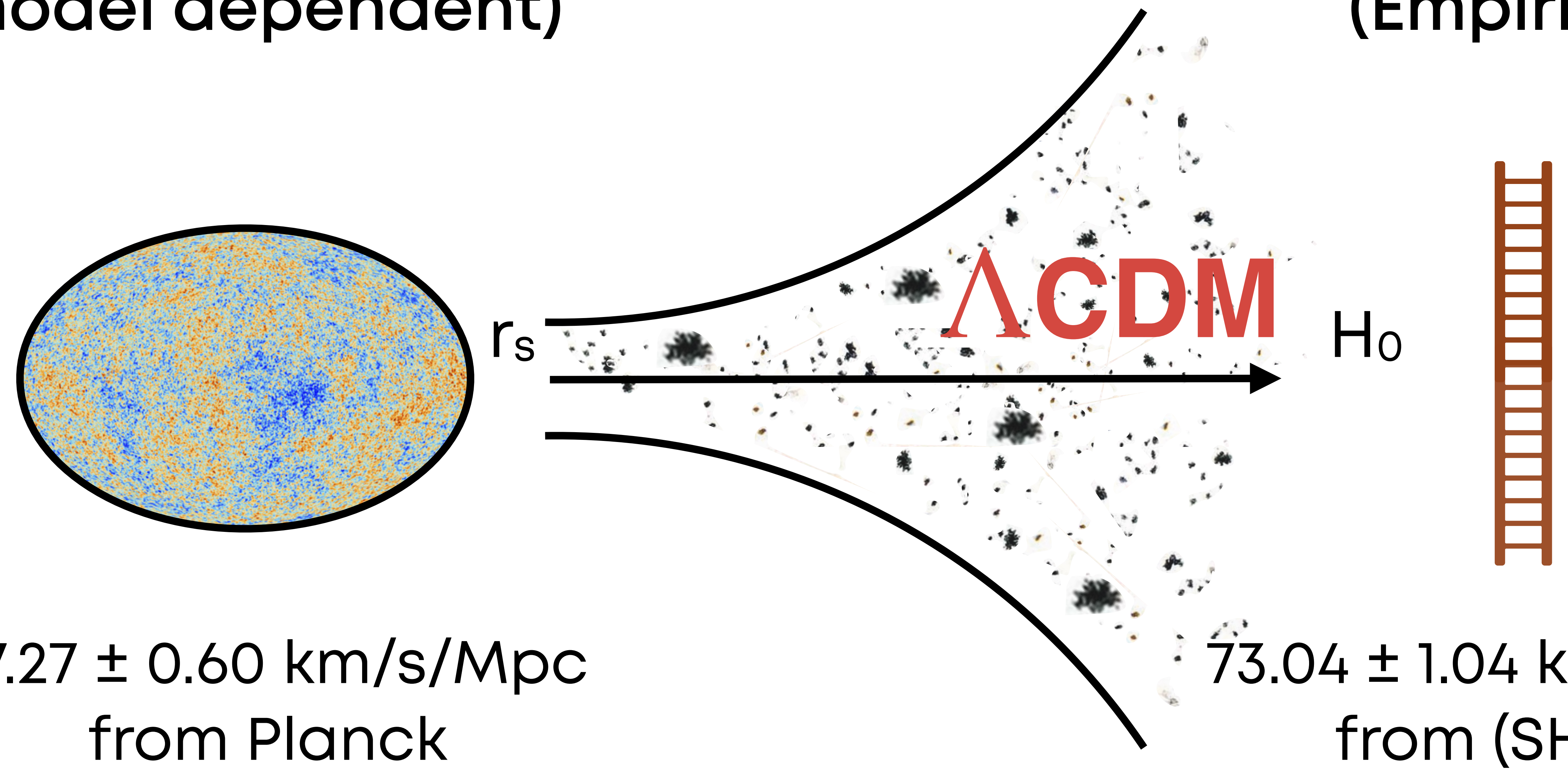
(Empirical)



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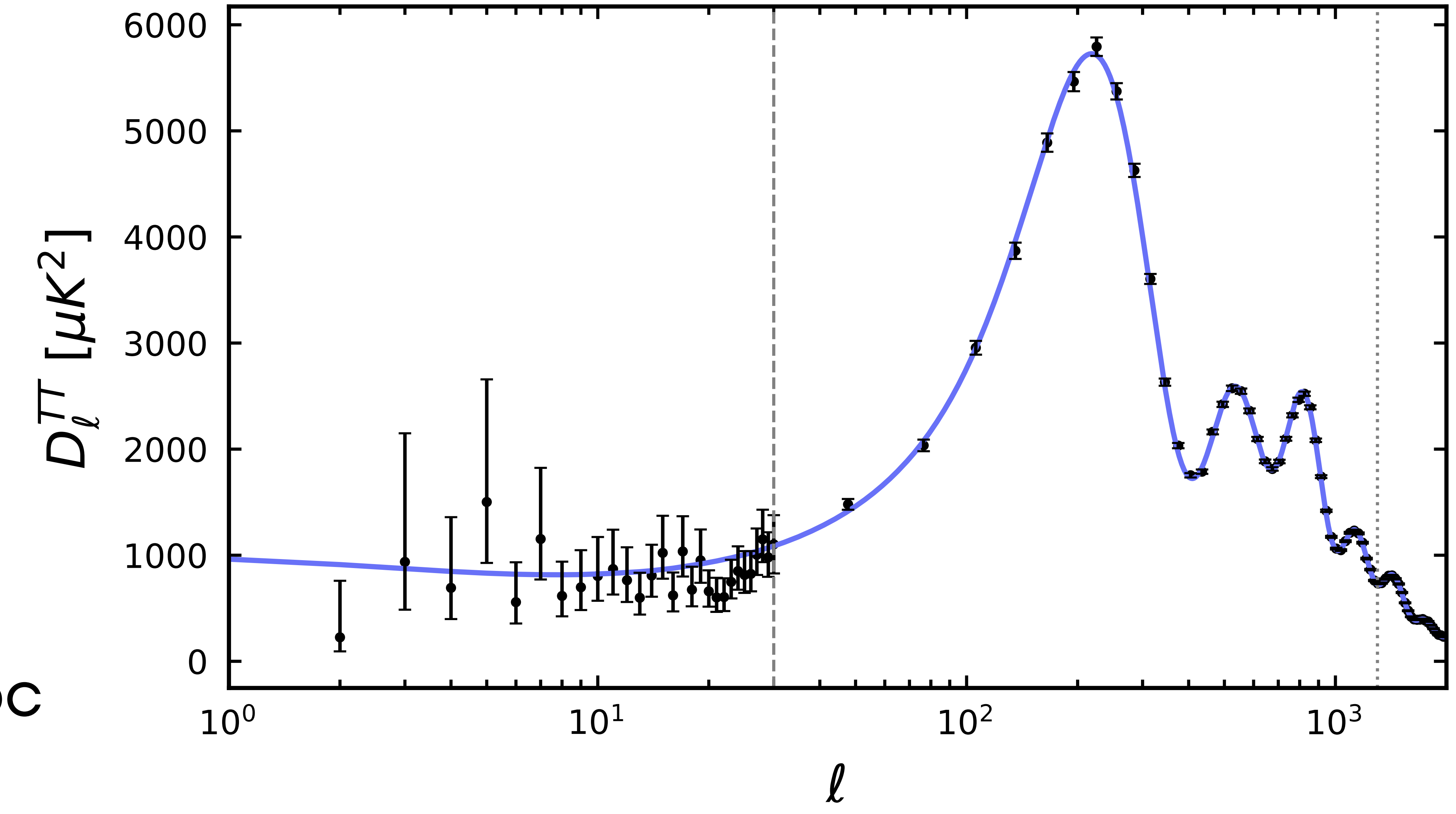


- Infer H_0 from the cosmological distance ladder
- Based on local distance measurements and astrophysical observables/calibrations

But how do we measure distances?

The distance duality relation

D_A from BAO

D_L from SN1a

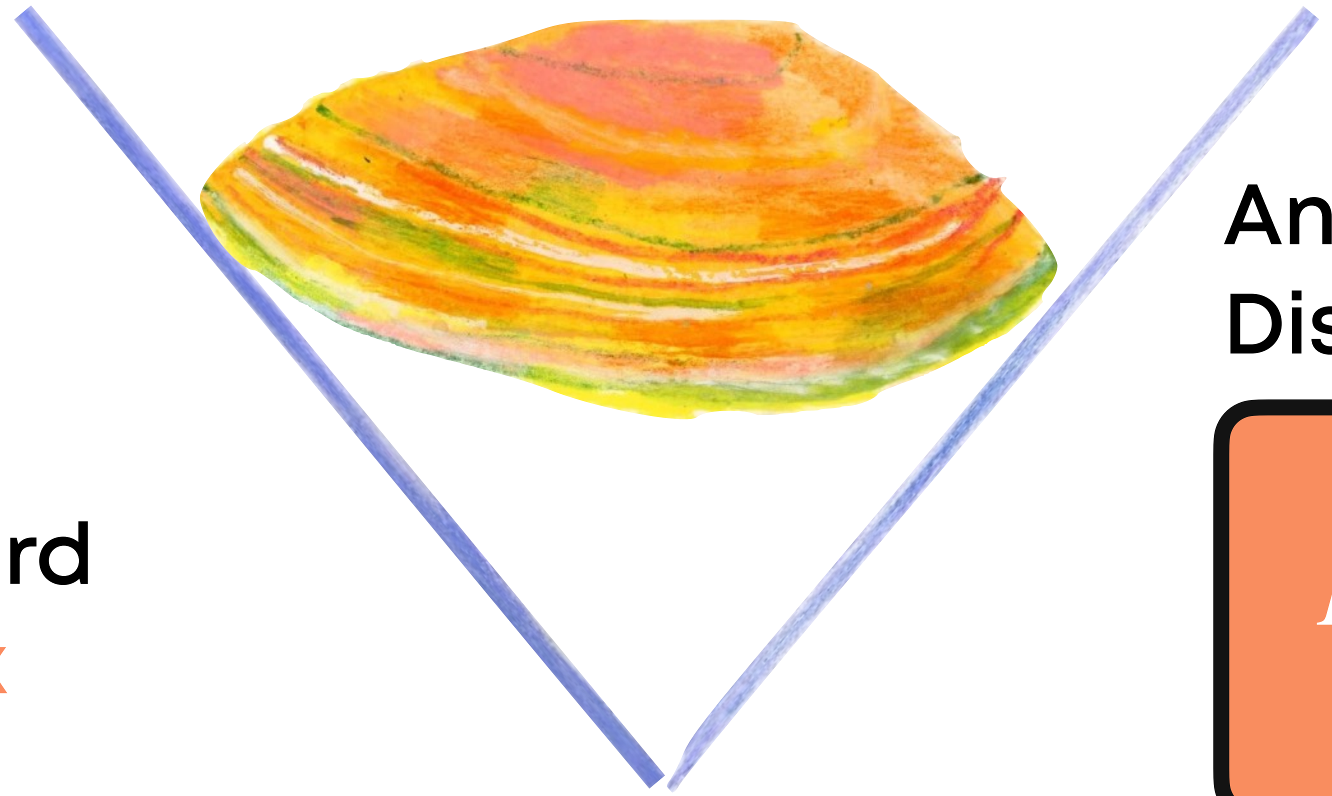
Redshift z

D_A from BAO

D_L from SN1a

Redshift z

$\Delta\theta_1$



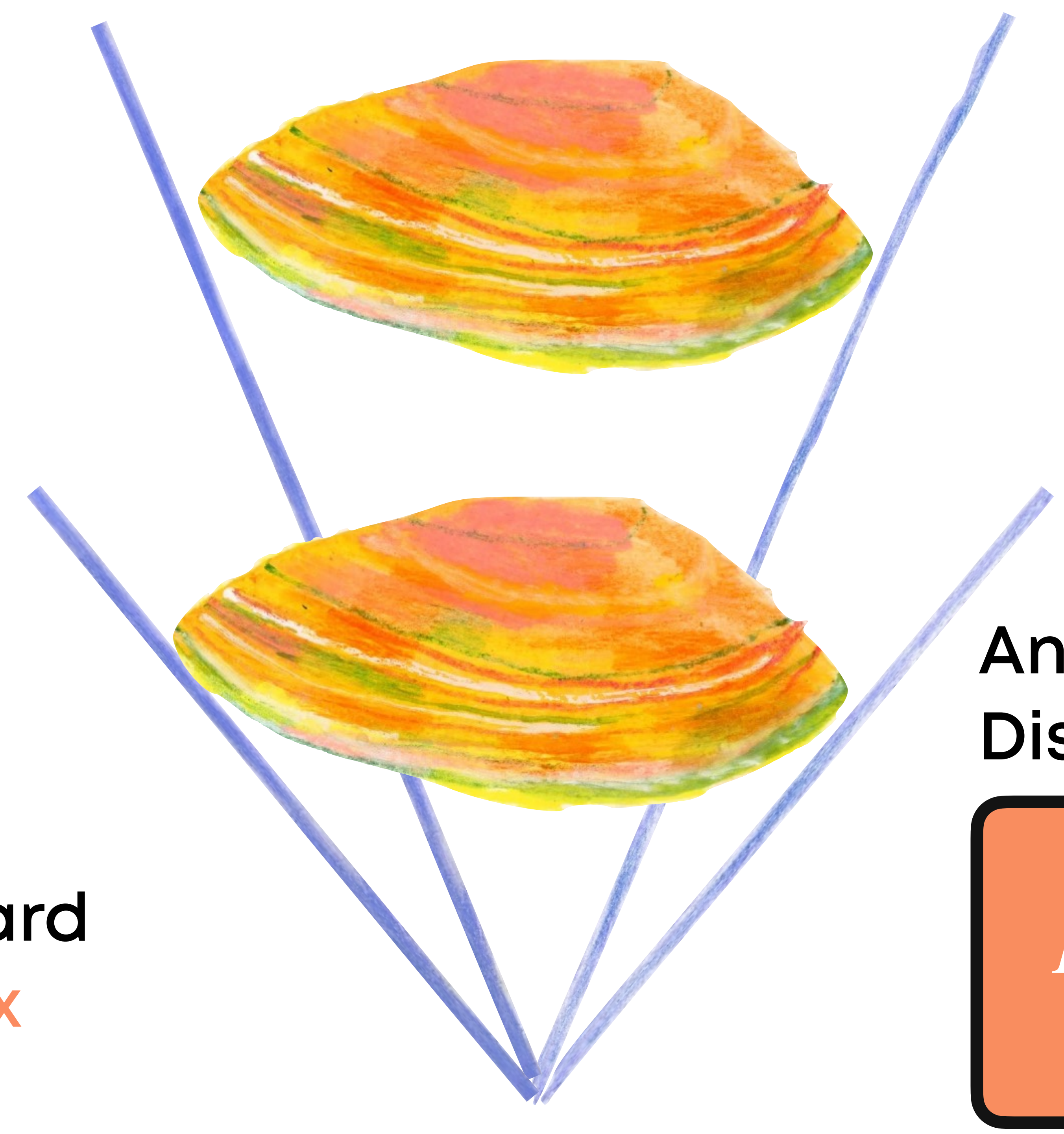
Standard ruler Δx

Angular Diameter Distance

$$D_A(z) = \frac{\Delta x}{\Delta\theta}$$

D_A from BAO

D_L from SN1a



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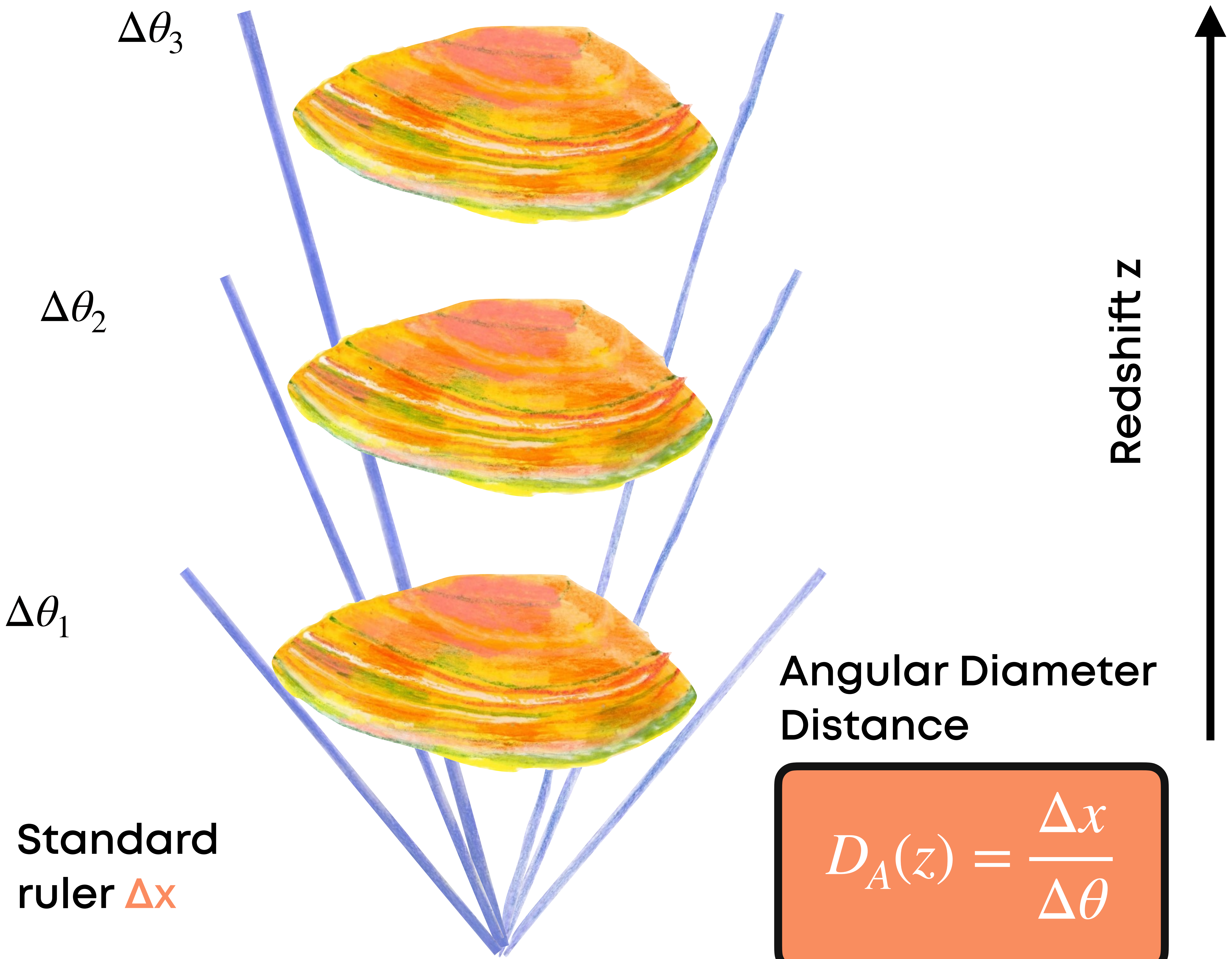

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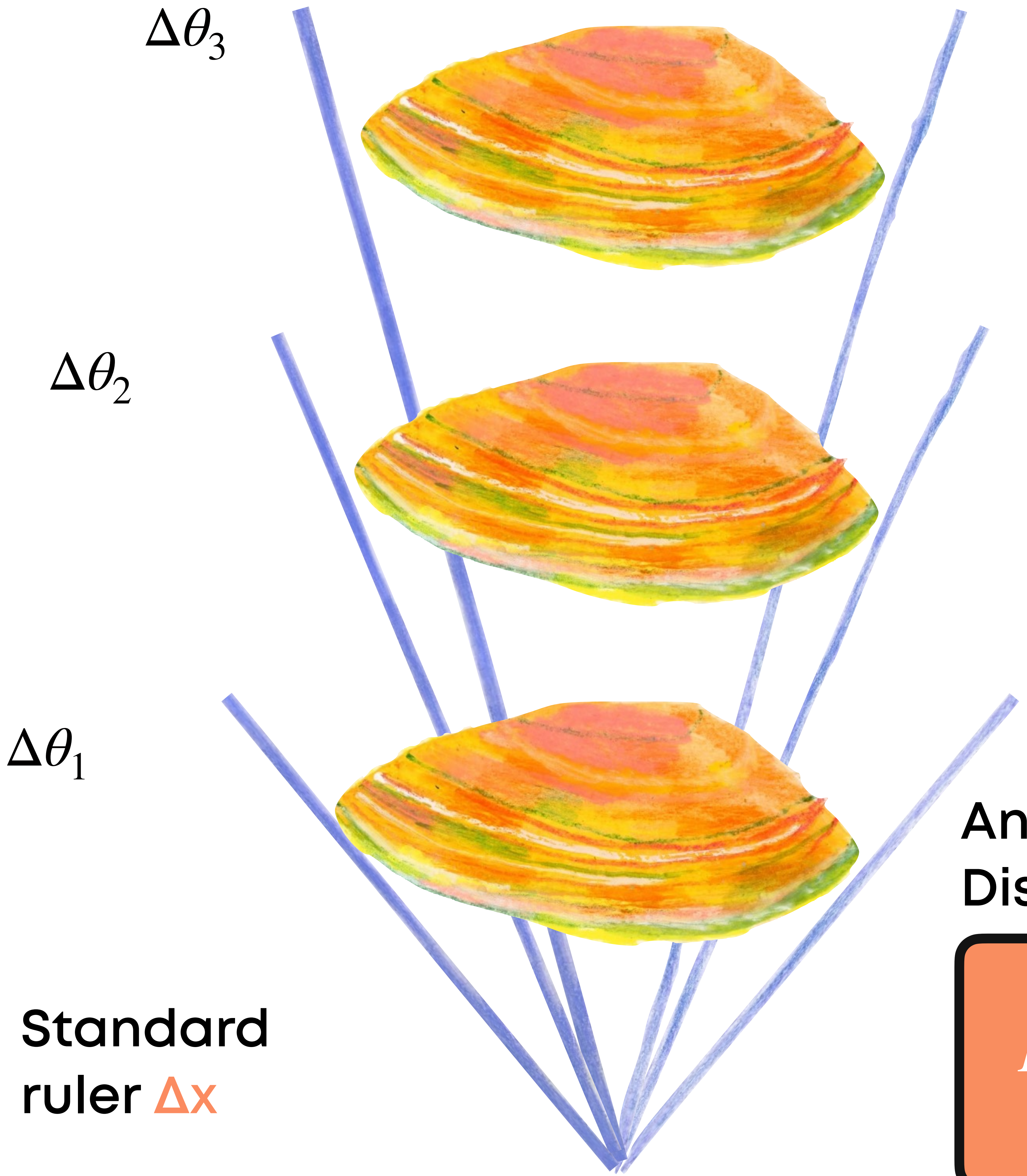


Standard ruler Δx

Redshift z

Angular Diameter Distance

$$D_A(z) = \frac{\Delta x}{\Delta \theta}$$

Luminosity Distance

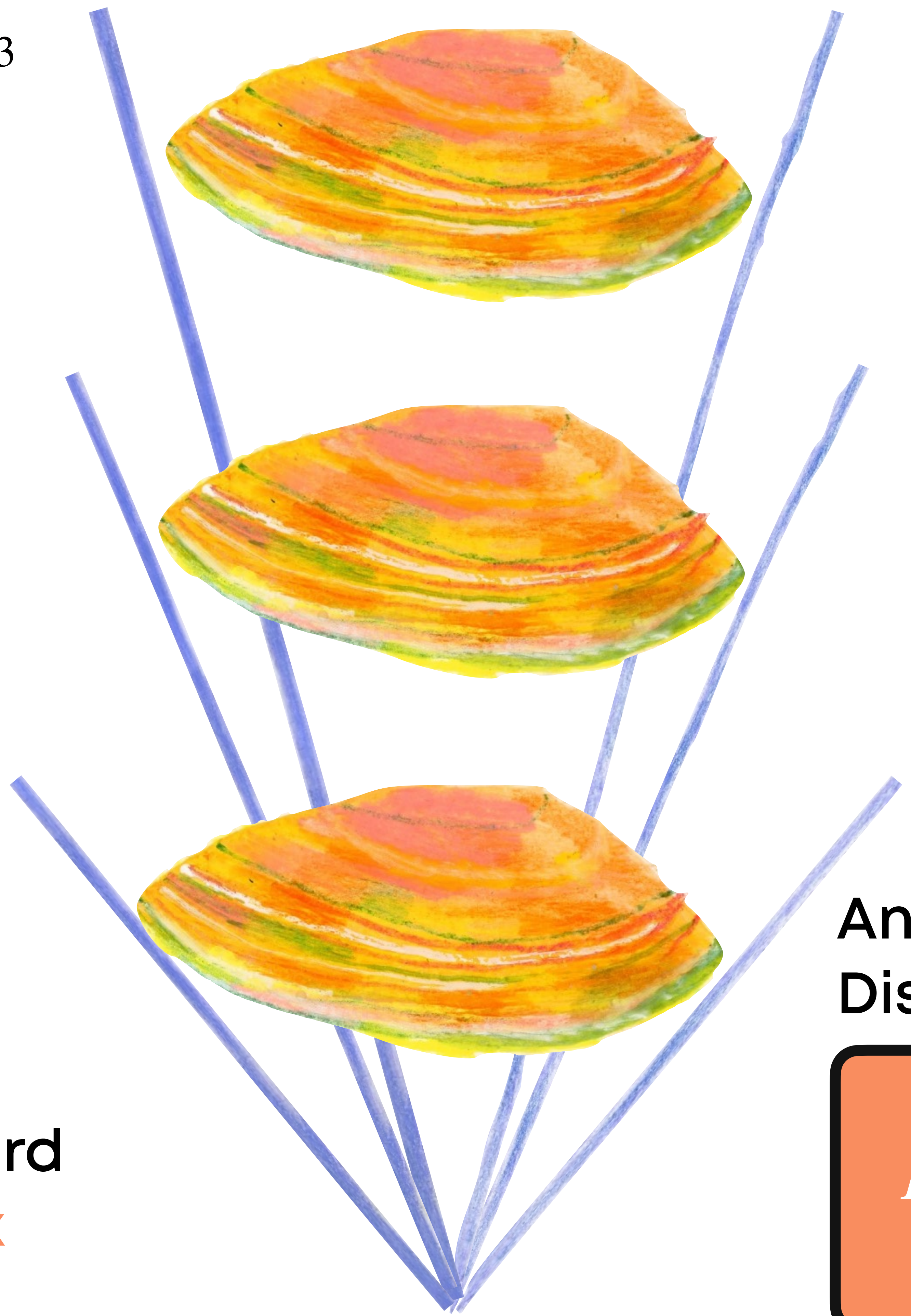
$$D_L^2(z) \equiv \frac{L}{4\pi F}$$

F_1

Standard candle L

D_A from BAO

D_L from SN1a



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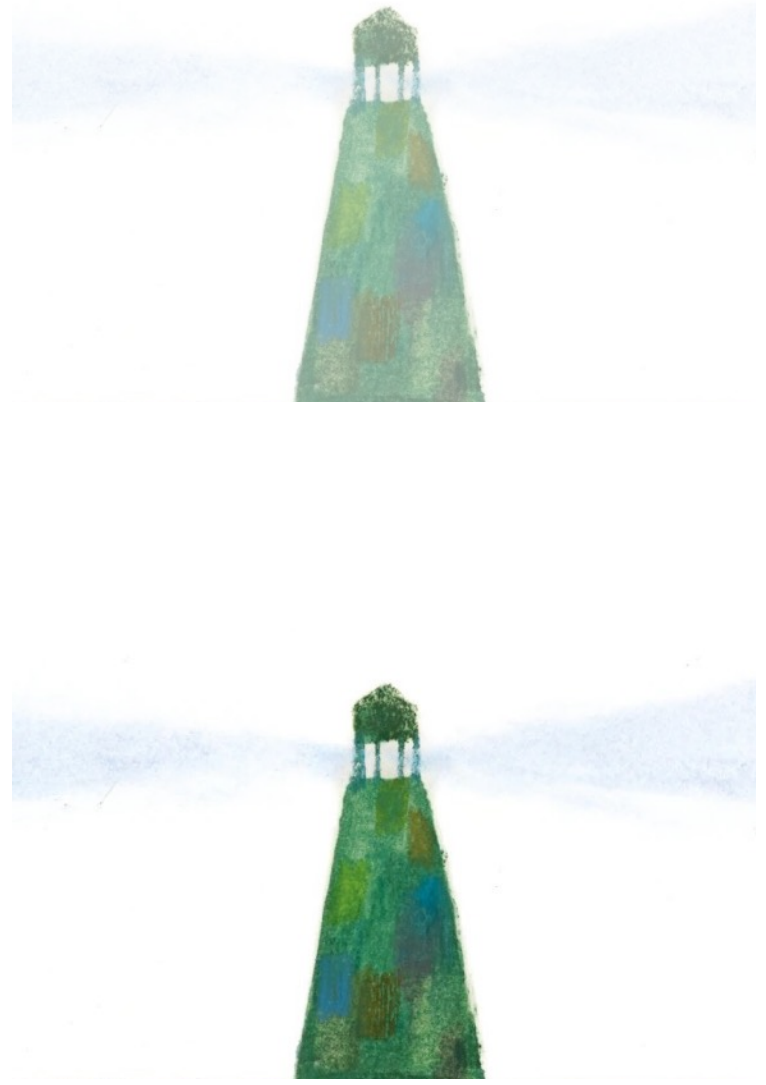
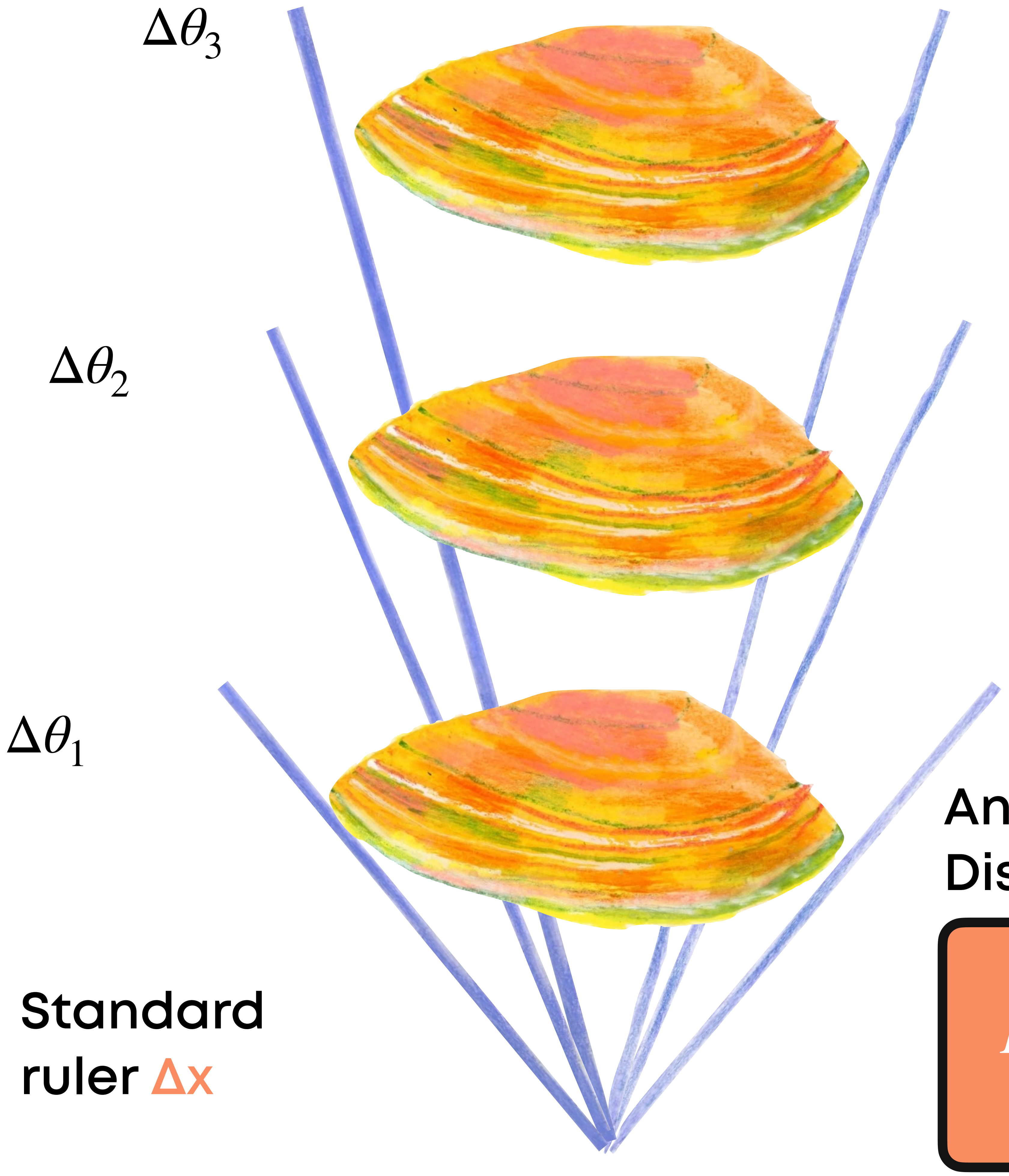


Standard candle L

D_A from BAO

D_L from SN1a



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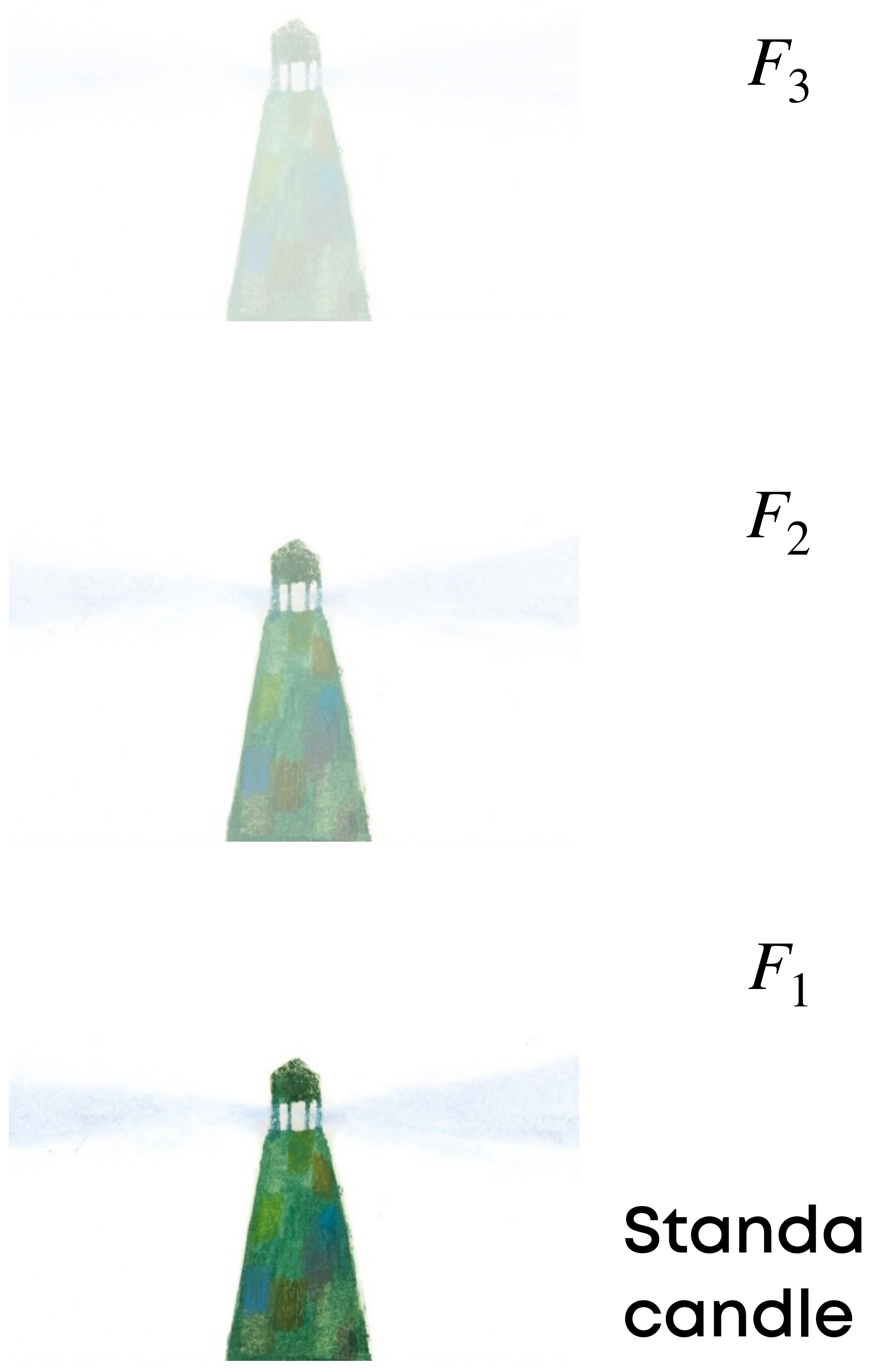


Standard candle L

Distance Duality Relation (DDR)

D_A from BAO

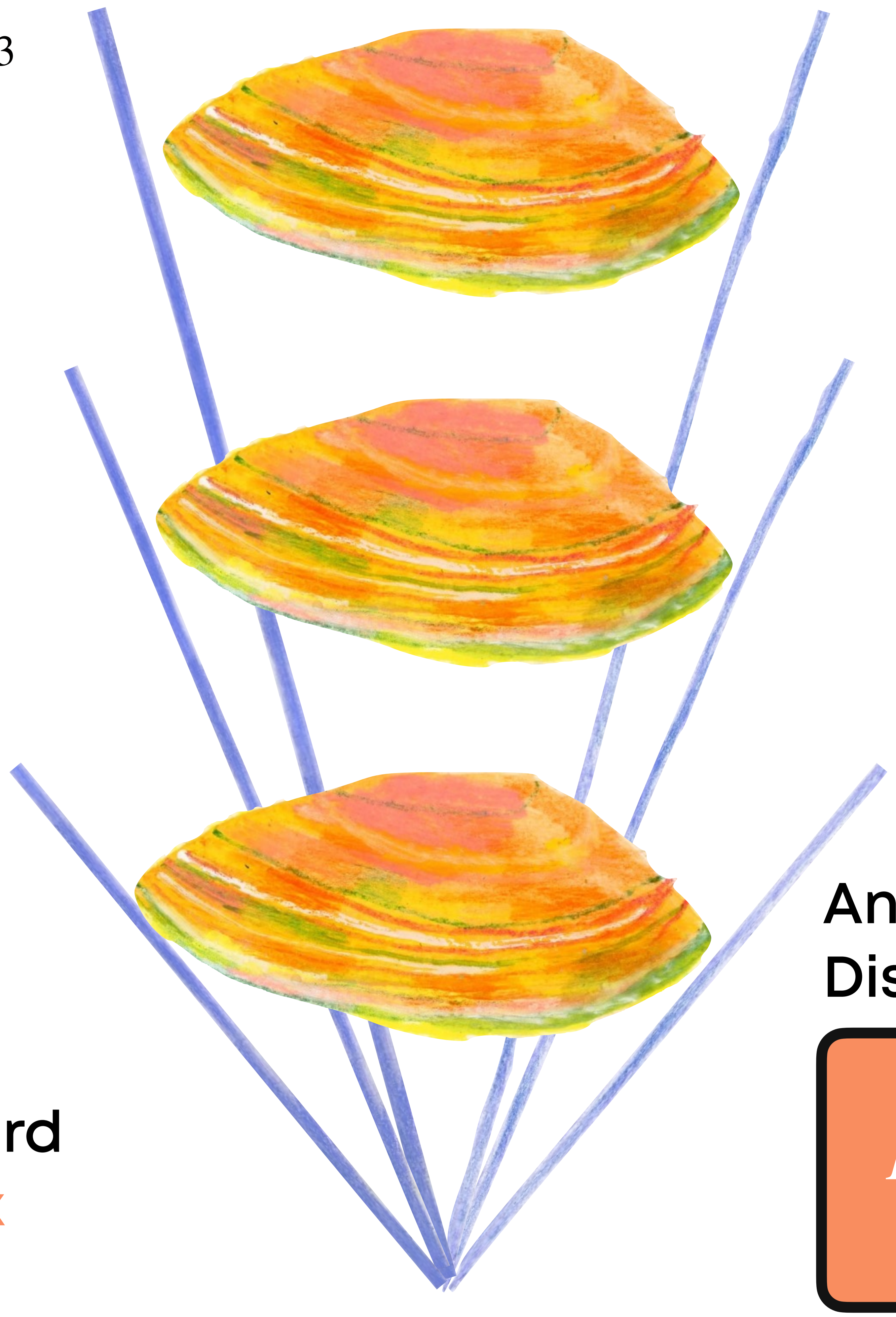
$$D_L(z) = (1+z)^2 D_A(z)$$

D_L from SN1a

$\Delta\theta_3$

$\Delta\theta_2$

$\Delta\theta_1$



Standard ruler Δx

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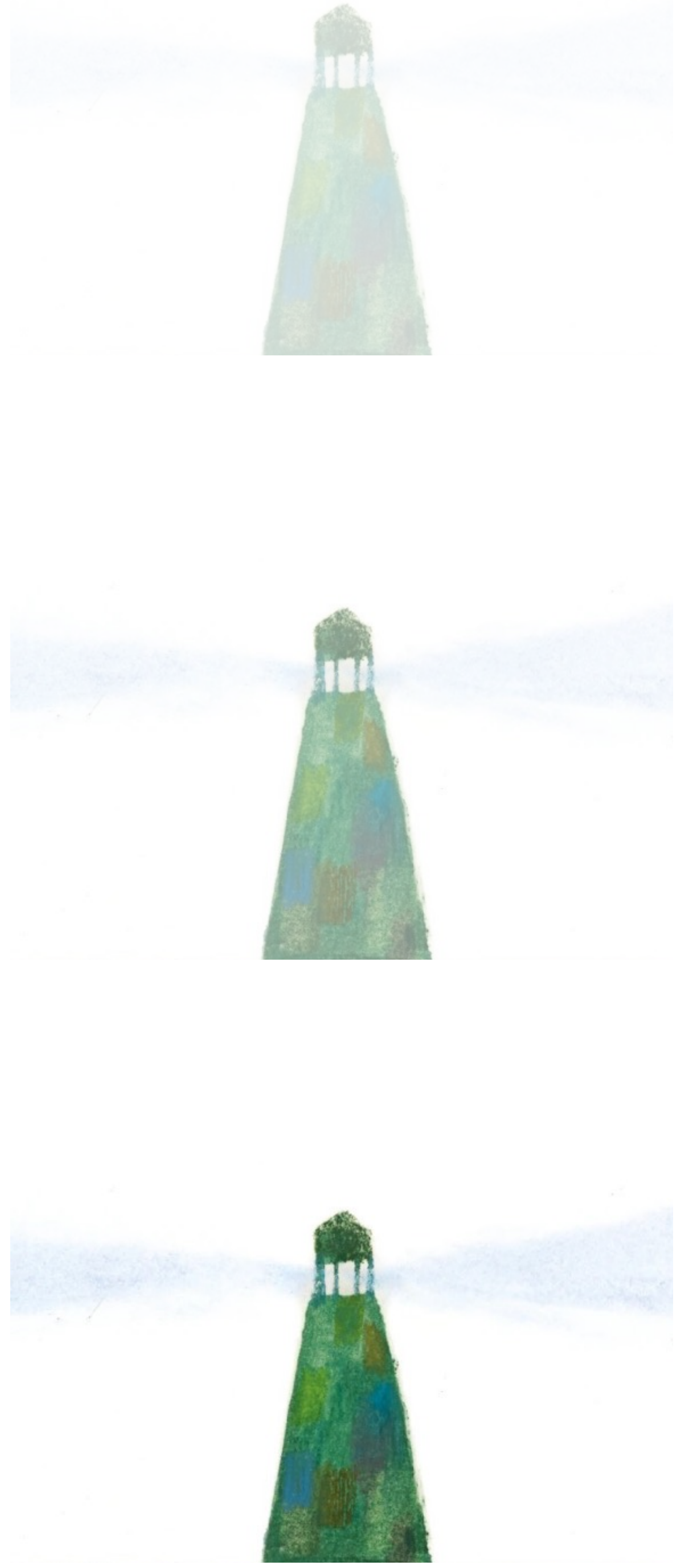


F_3

F_2

F_1

Standard candle L

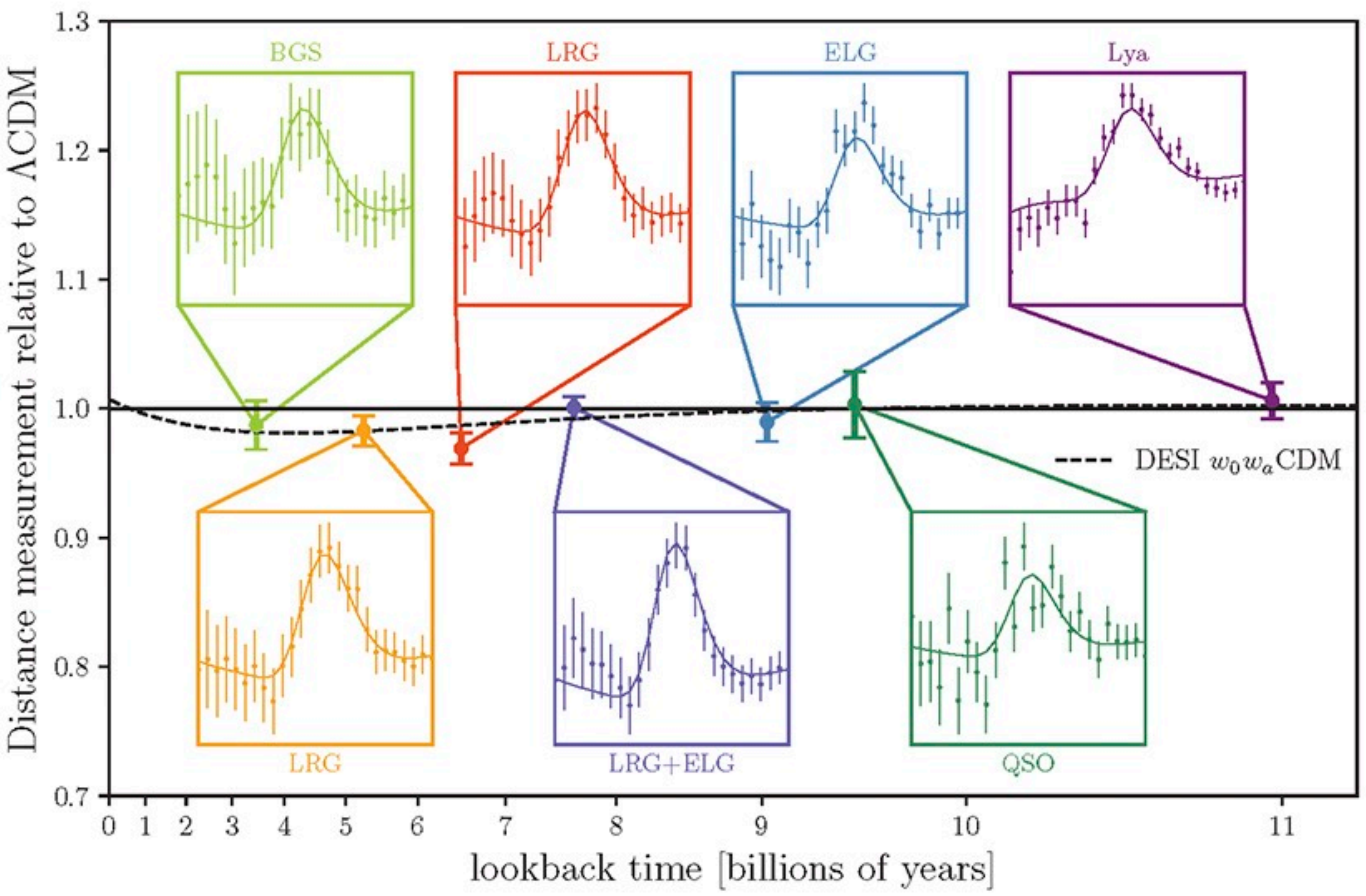
Distance Duality Relation (DDR)

D_A from BAO by DESI

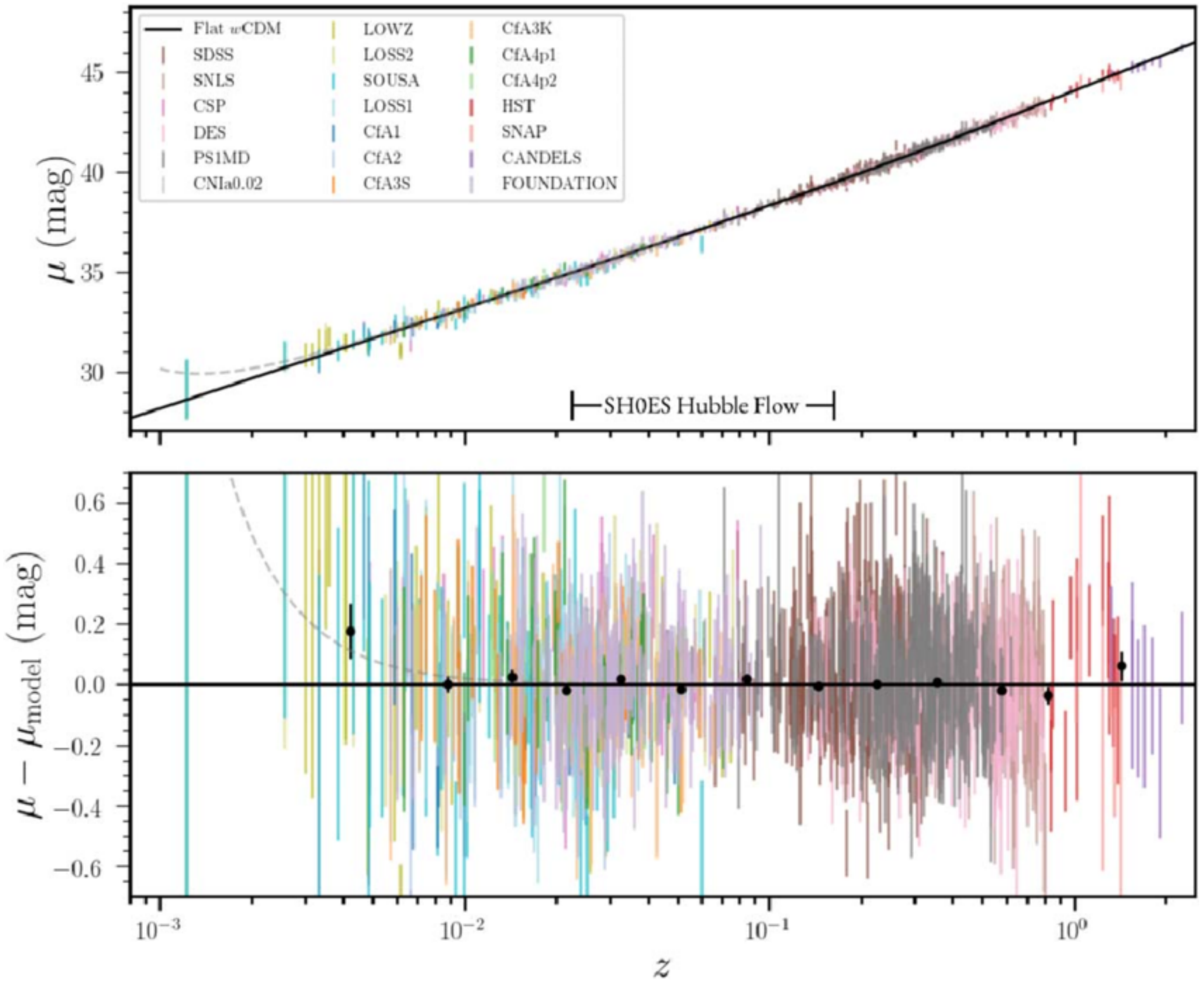
$$D_L(z) = (1+z)^2 D_A(z)$$

[I. M. H. Etherington (1933)]

D_L from SN1a by Pantheon+



[DESI Collaboration 2024, arxiv:2404.03002]



[The Pantheon+ Analysis 2022, arxiv:2202.04077]

Hubble tension or distance tension?

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BAO: $\theta_d(z) = \dfrac{r_s(z_{\rm drag})}{D_A(z)}$

From *Planck*: $r_s \sim 147$ Mpc:

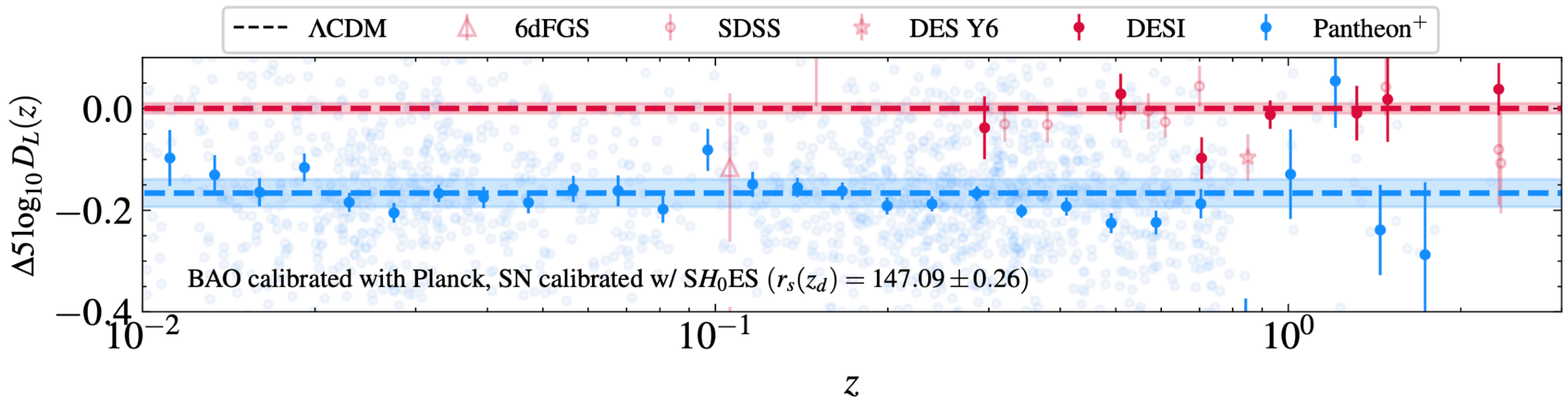
[Aghanim et al.: Astron.Astrophys. 641 (2020) A6]

$D_A(z)$ and $D_L(z)$ are incompatible!

From SH0ES: $M_b \sim -19.25$

[Riess et. al: Astrophys. J. Lett. 934 (2022) 1 L7]

SN1a: $m(z) = 5\log_{10}\left(D_L(z)\right) + M_b$



[Poulin et al.: arXiv: 2407.18292]

[Camarena et al.: arXiv: 2101.08641]
[Efstathiou: arXiv: 2103.08723]
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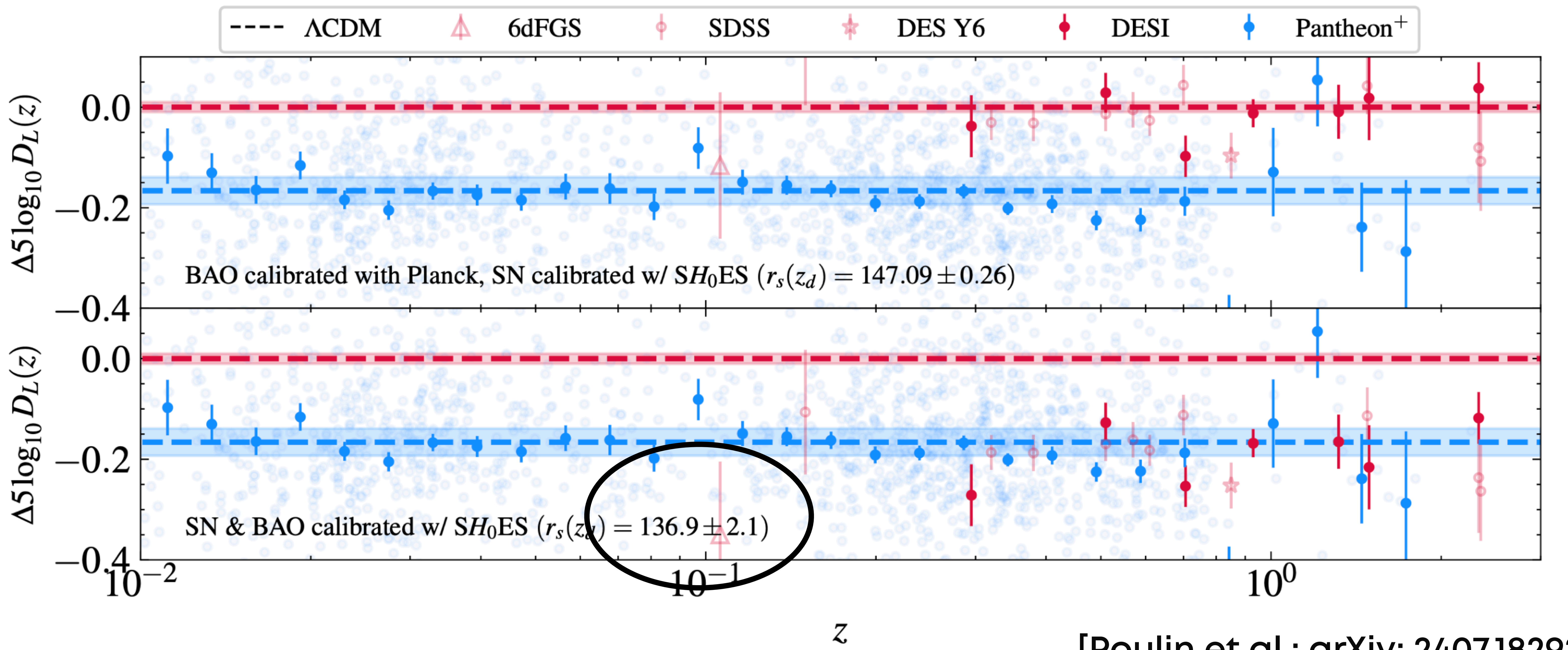

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Bring the data sets together:

- Change calibrators, e.g. change r_s (constant overall shift)

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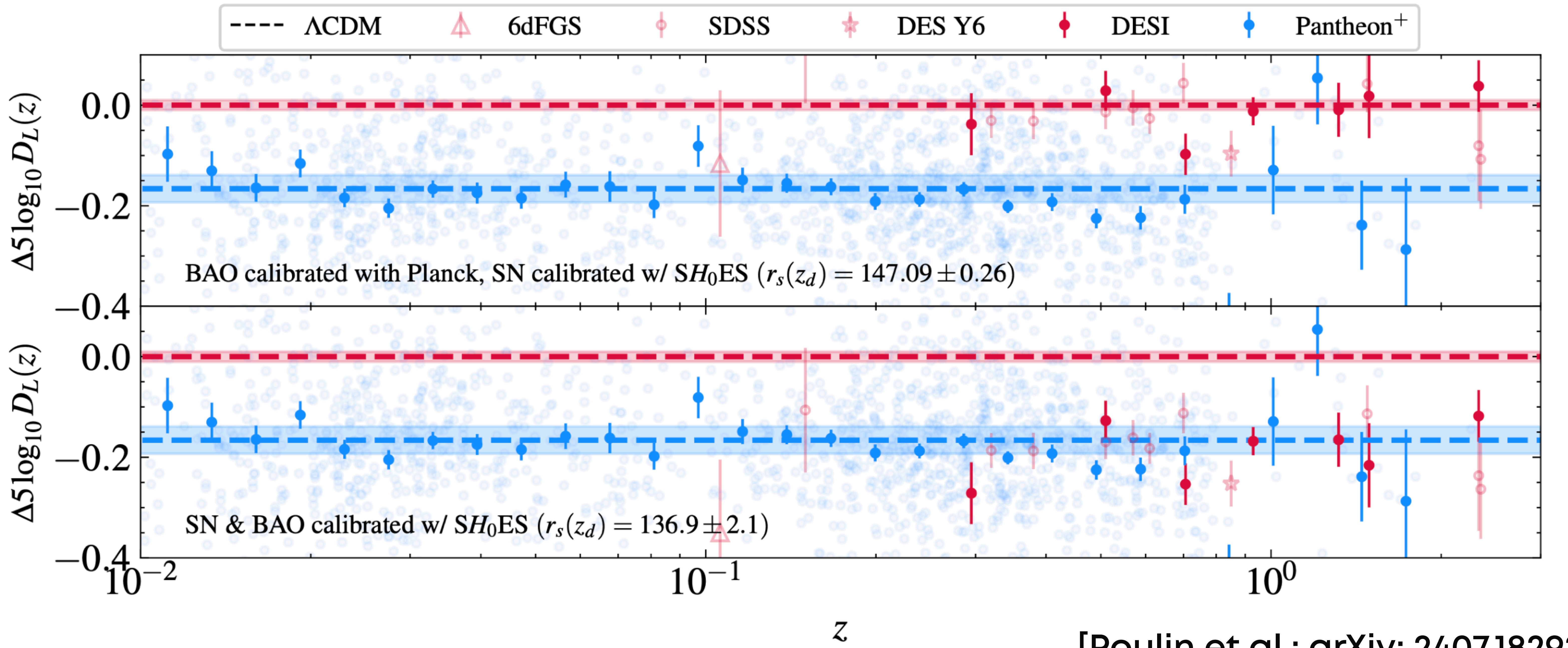


[Poulin et al.: arXiv: 2407.18292]

Bring the data sets together:

- Change calibrators, e.g. change r_s (constant overall shift)
- Break the "distance duality relation" (possible redshift dependance)

[Camarena et al.: arXiv: 2101.08641]
[Efstathiou: arXiv: 2103.08723]
[Raveri: arXiv: 2309.06795]
[Tutusaus et al.: arXiv: 2311.16862]

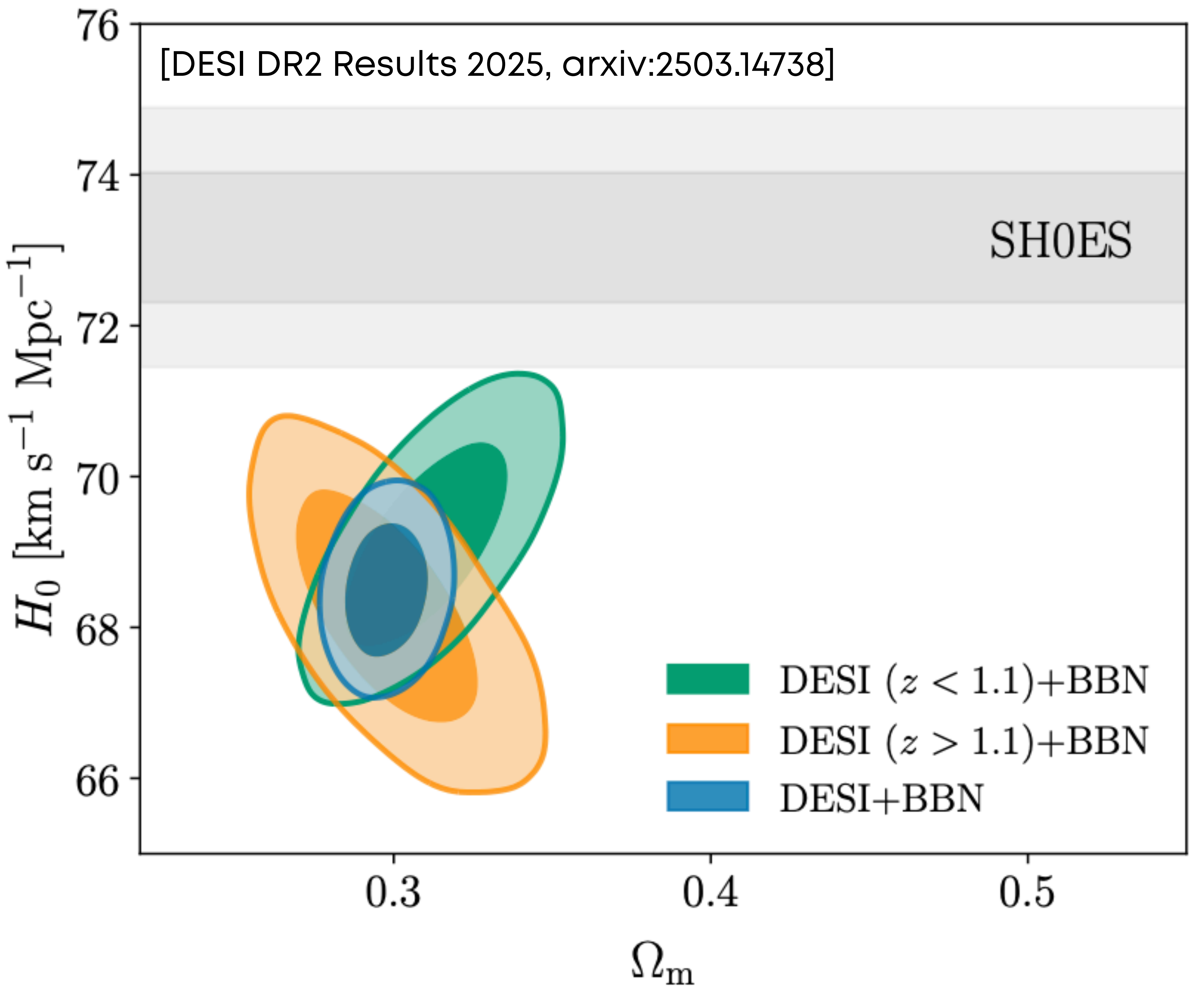
The DDR and evidence for Dynamical Dark Energy

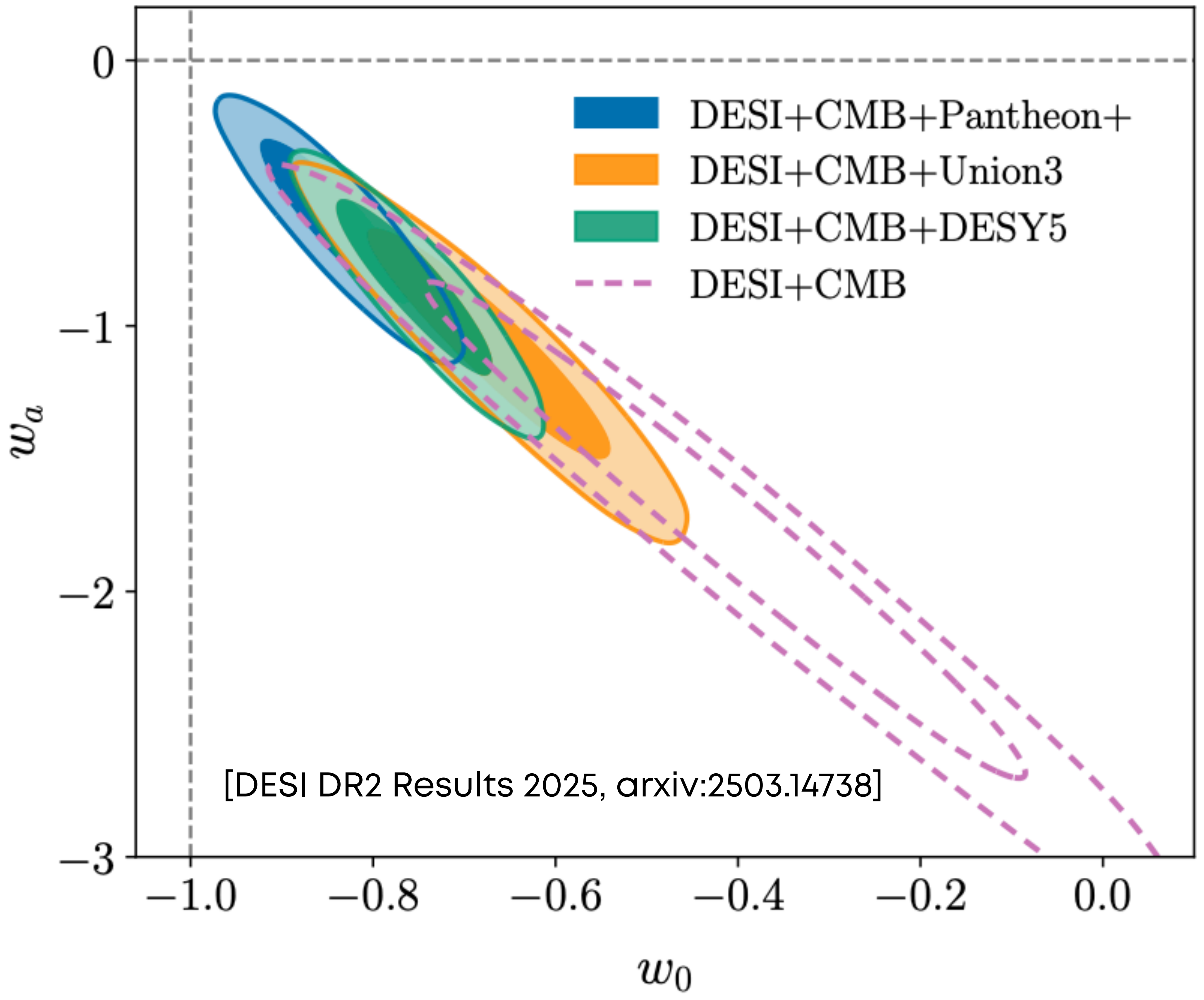
Based on: [E. M. Teixeira, W. Giarè, N. B. Hogg, T. Montandon, A. Poudou, and V. Poulin: arxiv:2504.10464]

The H_0 tension at late times

- Tension of more than 3σ with SH0ES
- Hints of dynamical DE in DESI data

$$\dot{\rho}_{\mathrm{DE}} + 3H\rho_{\mathrm{DE}}(1+w) = 0, \quad w = w_0 + w_a(1-a)$$





But how?

Reconciling the cosmological distances between DESI BAO and Pantheon+ SN

$$\eta(z) = \frac{D_L(z)}{(1+z)^2 D_A(z)}$$

- DDR is assumption of ΛCDM - holds for metric theories of gravity with photons travelling on null geodesics + their number conservation
- Violation encoded in $\eta(z)$: e.g. photons interacting with BSM particles or astrophysical absorption/opacity
- Proof of concept: effect of geometrical breaking of DDR for SN and BAO (no thermal evolution of CMB)
- If $\eta(z)$ is just a constant then we are probably dealing with calibration issues [Poulin et al.: arXiv: 2407.18292]
- Is there evidence for more than 1 dof and/or redshift dependence? Mechanisms and physical implications

Breaking the DDR

Reconciling the cosmological distances between DESI BAO and Pantheon+ SN

$$\eta(z) = \frac{D_L(z)}{(1+z)^2 D_A(z)}$$

1. Can simple phenomenological parameterisations of DDR violation in a ΛCDM background cosmology reduce/eliminate the tension between calibrated SNIa and BAO?

Breaking the DDR

Reconciling the cosmological distances between DESI BAO and Pantheon+ SN

1. Can simple phenomenological parameterisations of DDR violation in a ΛCDM background cosmology reduce/eliminate the tension between calibrated SNIa and BAO?

2. Is there evidence for a violation of the DDR that changes over cosmic history?

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Breaking the DDR

Reconciling the cosmological distances between DESI BAO and Pantheon+ SN

$$\eta(z) = \frac{D_L(z)}{(1+z)^2 D_A(z)}$$

1. Can simple phenomenological parameterisations of DDR violation in a ΛCDM background cosmology reduce/eliminate the tension between calibrated SNIa and BAO?

2. Is there evidence for a violation of the DDR that changes over cosmic history?

3. Can a DDR violation alter the preference for dynamical dark energy observed in the combination of current BAO and SNIa data?

Data Sets

- **Pantheon-plus (SN):** measurements of $\mu(z, D_L)$ from spectroscopically detected Type Ia supernovae in the redshift range $0.001 < z < 2.26$ [Brout et al.: ApJ 938 110 (2022)]
- **DESI Y1 BAO:** BAO measurements of $H(z)$ and $D_A(z)$ in the redshift range $z \sim 0.1 - 4.1$ [Adame et al.: arxiv:2404.03002]
- **SH0ES** prior on MB: $M_B \sim -19.25$ [Riess et. al: Astrophys. J. Lett. 934 (2022) 1 L7]
- **Planck 2018 CMB** data: (high-ℓ TTTEEE 'Plik-lite', 'Plik' low-ℓ TT and EE) [Aghanim et al.: Astron.Astrophys. 641 (2020) A5]

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DISCLAIMER!

DDR breaking
in SN and not in CMB
(late-Universe physics/ systematics)

SN1a: $m(z) = 5\log_{10}\left(D_L(z)\right) + M_b$

BAO: $\theta_d(z) = \dfrac{r_s(z_{\rm drag})}{D_A(z)}$

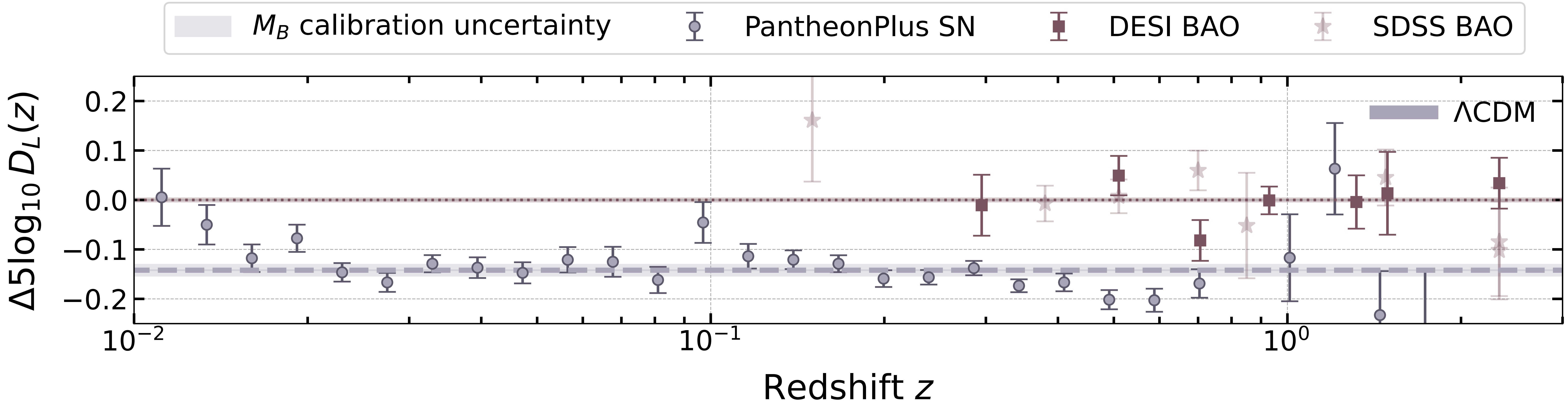
1. Breaking the DDR as a solution to the Hubble tension

Based on: [E. M. Teixeira, W. Giarè, N. B. Hogg, T. Montandon, A. Poudou, and V. Poulin: arxiv:2504.10464]

Combine the data

$$D_L(z) = (1+z)^2 D_A(z)$$

- In ΛCDM with no DDR violation the various BAO estimates of DL(z) are systematically larger than the SNIa estimates
- Tries to accommodate both data sets - bad overall fit reflecting tension

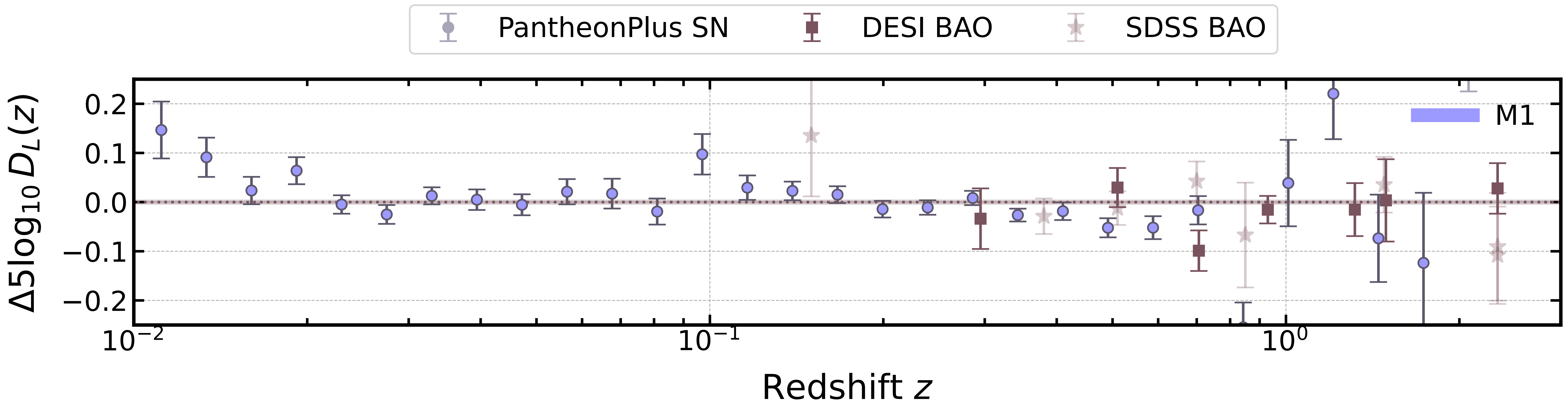


$$T = 5.7\sigma$$

M1 - Constant DDR

$$D_L(z) = (1+z)^2 D_A(z)\,(1+\alpha_0)$$

- A phenomenological violation of the DDR can indeed resolve the Hubble tension
- The model with the largest evidence is compatible with a simple rescaling of the calibration of either SNIa or BAO
- Reinforces the interpretation of the Hubble tension as a 'cosmic calibration tension'

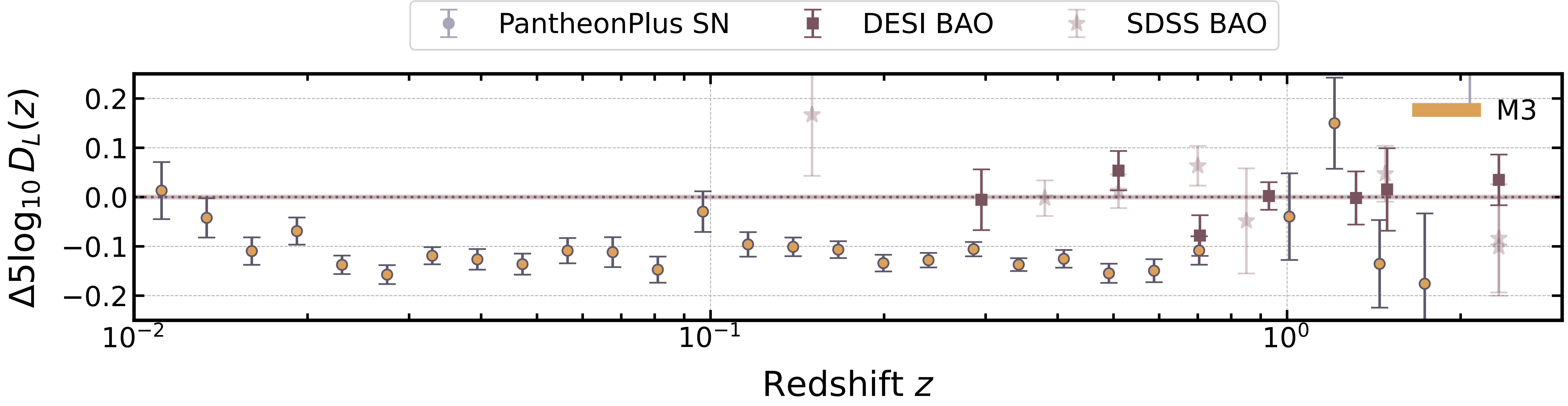


$$T = 0.2\sigma$$

M3 - Exponent DDR

$$D_L(z) = (1+z)^2 D_A(z)\,(1+z)^{\alpha_0}$$

- A power-law provides a natural framework for describing DDR violations that may arise from a violation of photon number conservation
- For the case of CMB photons, this effect can be related to changes in the temperature-redshift relation $T(z) = T_0(1+z)$ [Ruchika, W Giarè, EMT, A. Melchiorri.: arXiv: 2505.02909]
- Cannot resolve the Hubble tension



$T = 5.0\sigma$

2. A DDR deviation or a change in calibration?

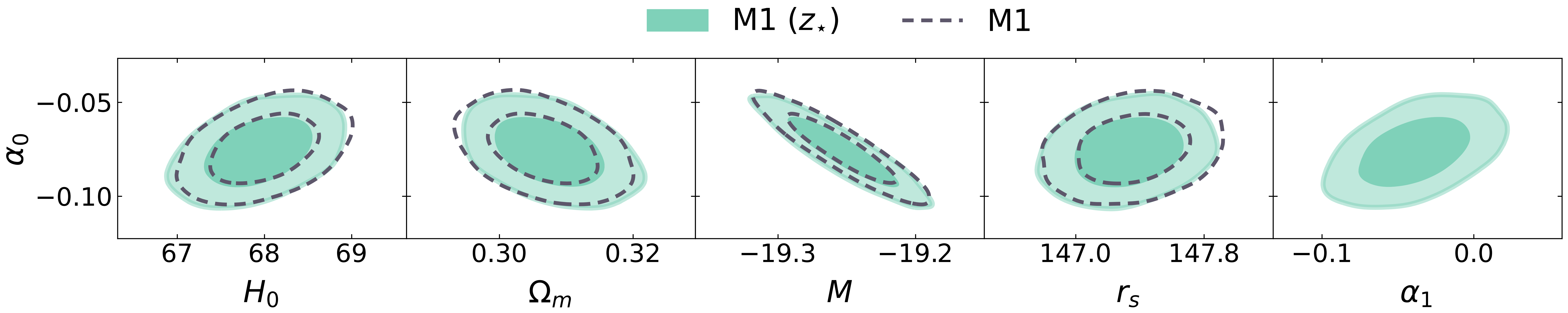
Based on: [E. M. Teixeira, W. Giarè, N. B. Hogg, T. Montandon, A. Poudou, and V. Poulin: arxiv:2504.10464]

M1(z*) - Double Constant DDR

$$D_L(z) = \begin{cases} (1+z)^2 D_A(z)\,(1+\alpha_0) & \text{if } z < 0.9 \\ (1+z)^2 D_A(z)\,(1+\alpha_1) & \text{if } z \geq 0.9 \end{cases}$$

- No evidence for extra degree of freedom for redshift dependance
- Very similar fit with one extra parameter

$$T = 0.08\sigma$$



M3(z*) - Double Exponent DDR

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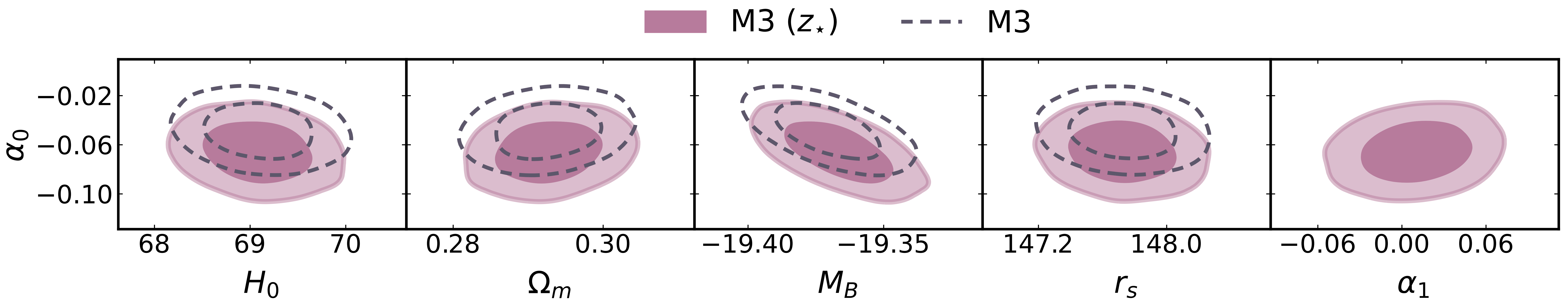
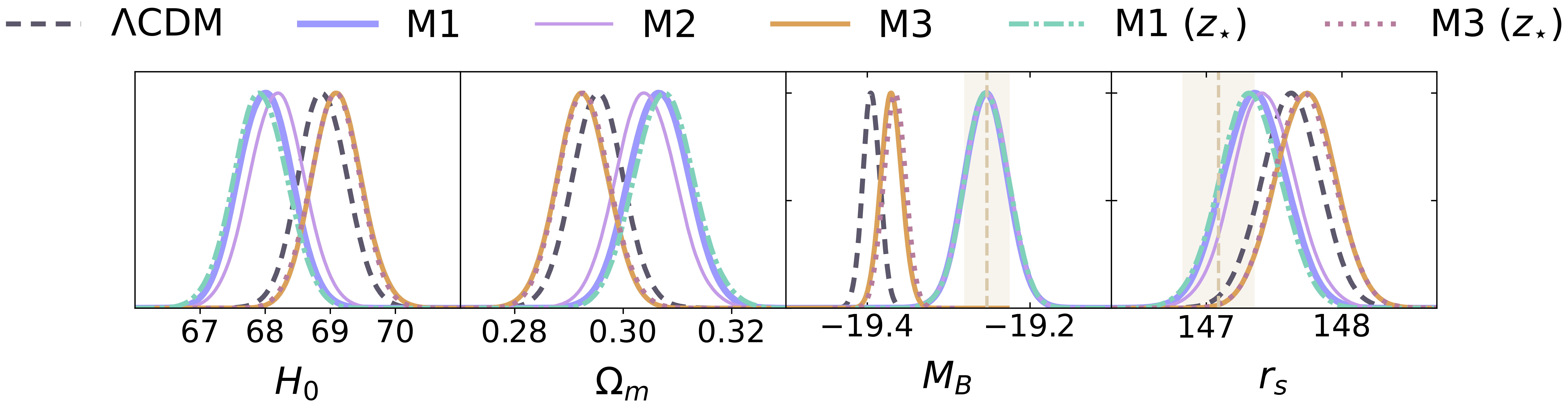
- No evidence for extra degree of freedom for redshift dependance
- Very similar fit with one extra parameter and no tension resolution

$$T = 4.8\sigma$$



Evidence for constant DDR

$$D_L(z) = 0.925\,(1+z)^2 D_A(z)$$



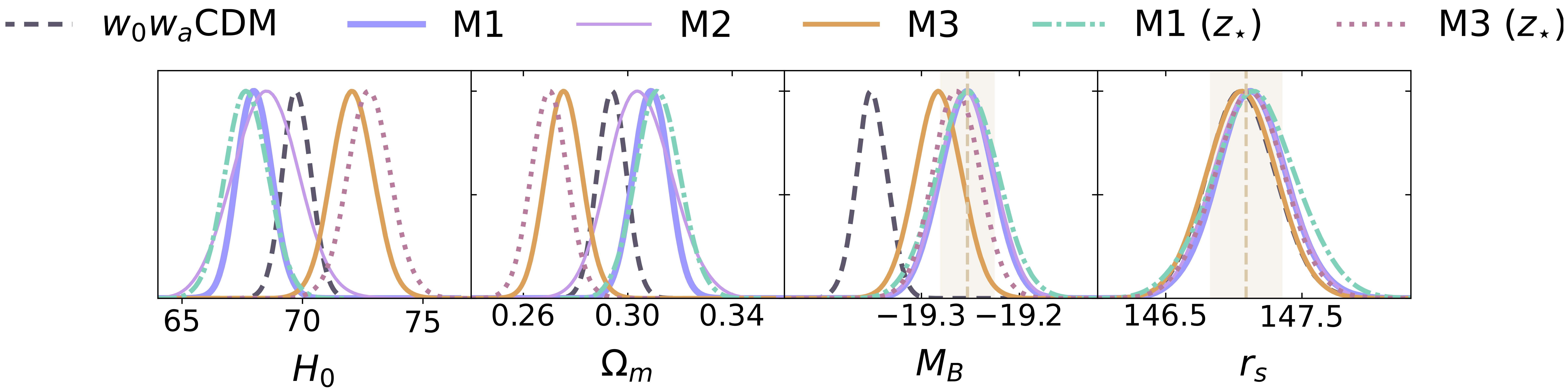
3. Degeneracy between DDR and dynamical dark energy

Based on: [E. M. Teixeira, W. Giarè, N. B. Hogg, T. Montandon, A. Poudou, and V. Poulin: arxiv:2504.10464]

Evidence for dynamical DE

$$\dot{\rho}_{\rm DE} + 3H\rho_{\rm DE}(1+w) = 0, \quad w = w_0 + w_a(1-a)$$

- The model that performs better is the double exponential DDR (M3(z*)) with phantom dark energy with $w \sim -1.155$ and a deviation in the DDR affecting the data at $z \lesssim 0.9$ with $\alpha_0 \sim -0.134$
- Definite preference over a constant deviation (M1) for both a ΛCDM or $w_0 w_a$CDM background



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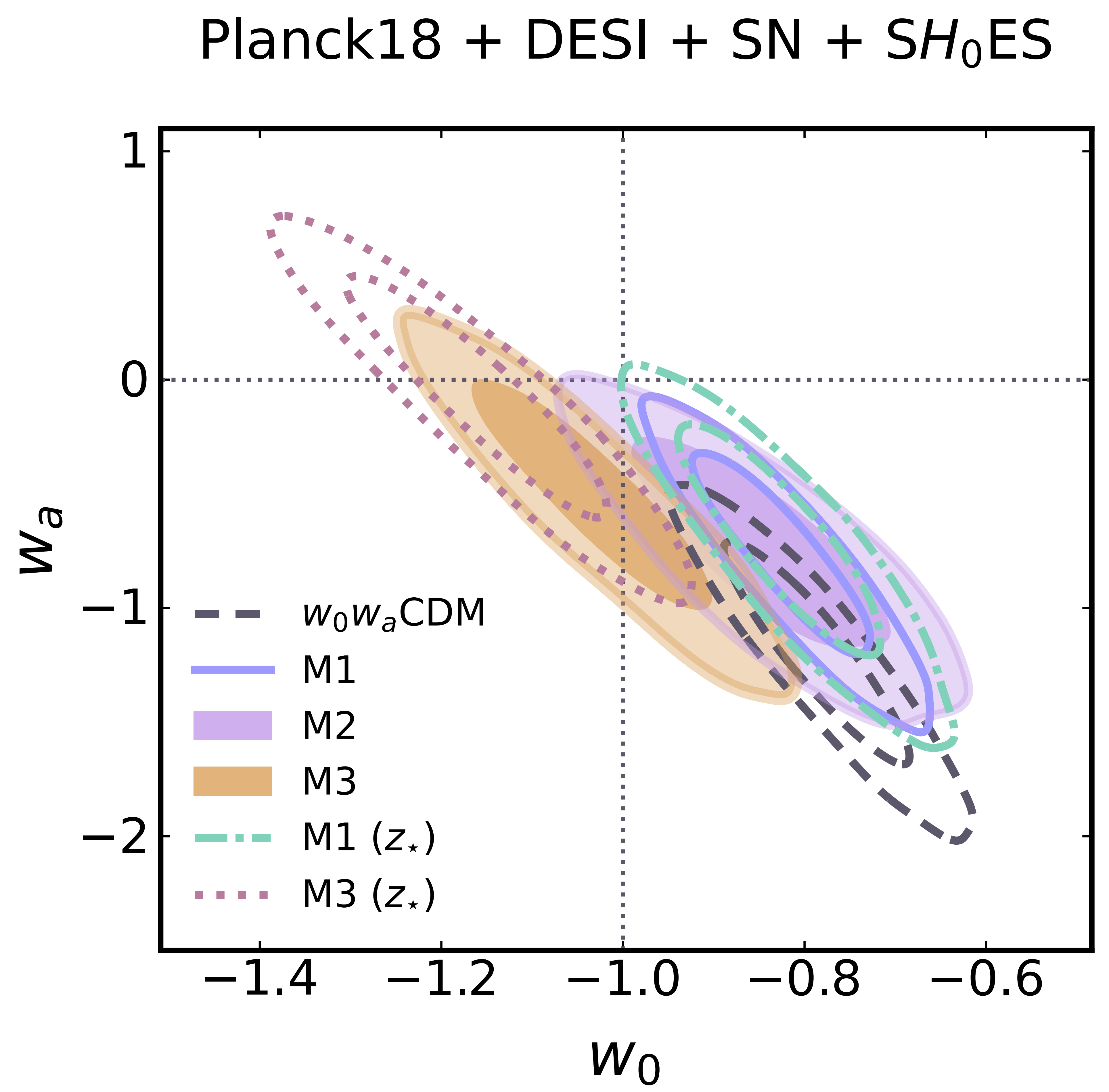
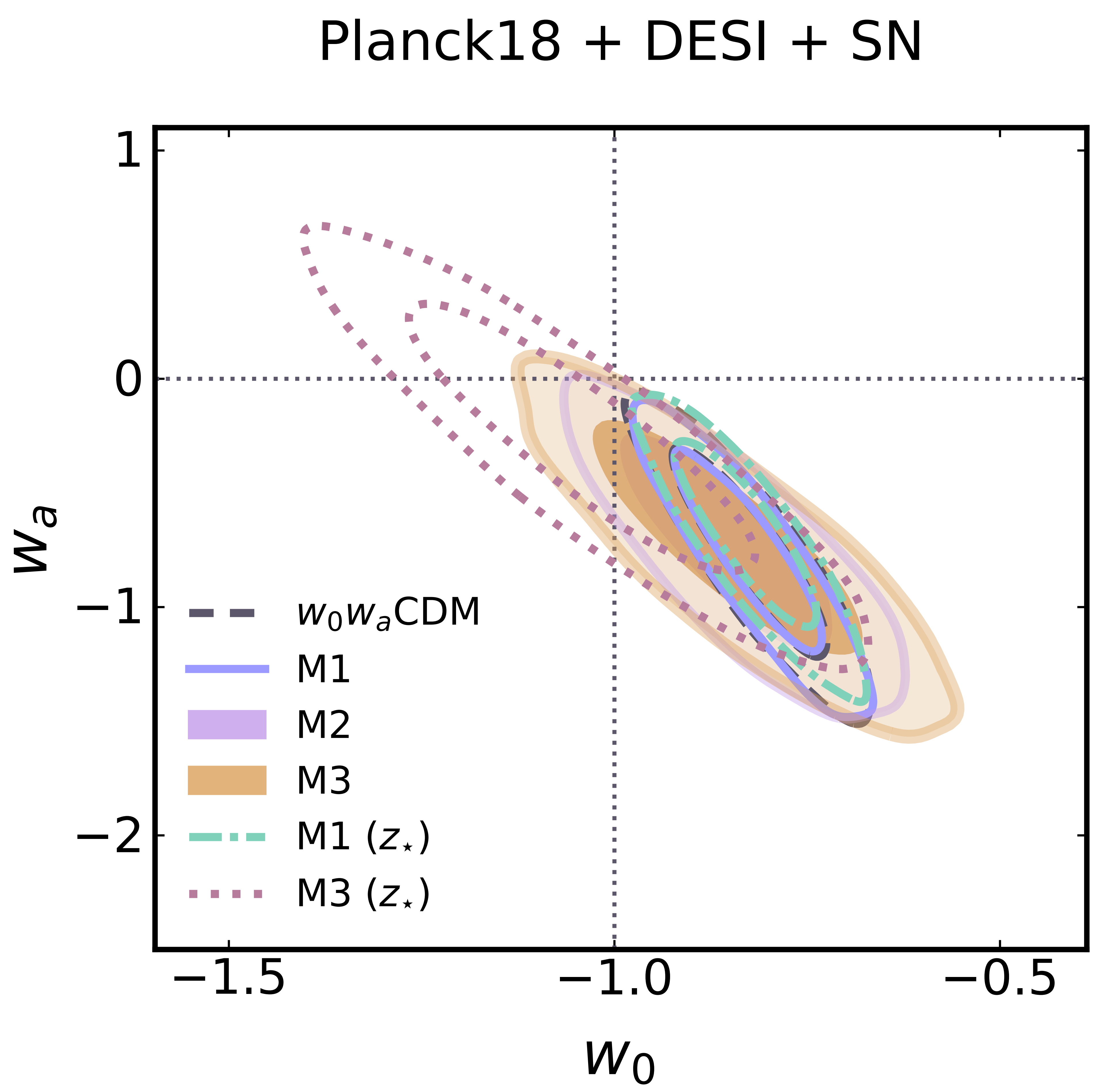
- Degeneracy between a violation of the DDR and a change in the background cosmology that affects the expansion history $H(z)/H_0$



Evidence for dynamical DE

$$\dot{\rho}_{\rm DE} + 3H\rho_{\rm DE}(1+w) = 0, \quad w = w_0 + w_a(1-a)$$

- Degeneracy between a violation of the DDR and a change in the background cosmology that affects the expansion history $H(z)/H_0$
- In the double exponent DDR (M3(z∗)), the preference for DDE vanishes without SH0ES (ΛCDM at ~ 1σ) and $H_0 = 72.77 \pm 0.91$ km/s/Mpc
- The preference for DDE can be interpreted as a break in the DDR, with a preference for a deviation occurring at $z < z_*$ (i.e., $\alpha_0 \neq 0, \alpha_1 \sim 0$)
- However: DESI DR2 finds evidence for DDE just from DESI+CMB



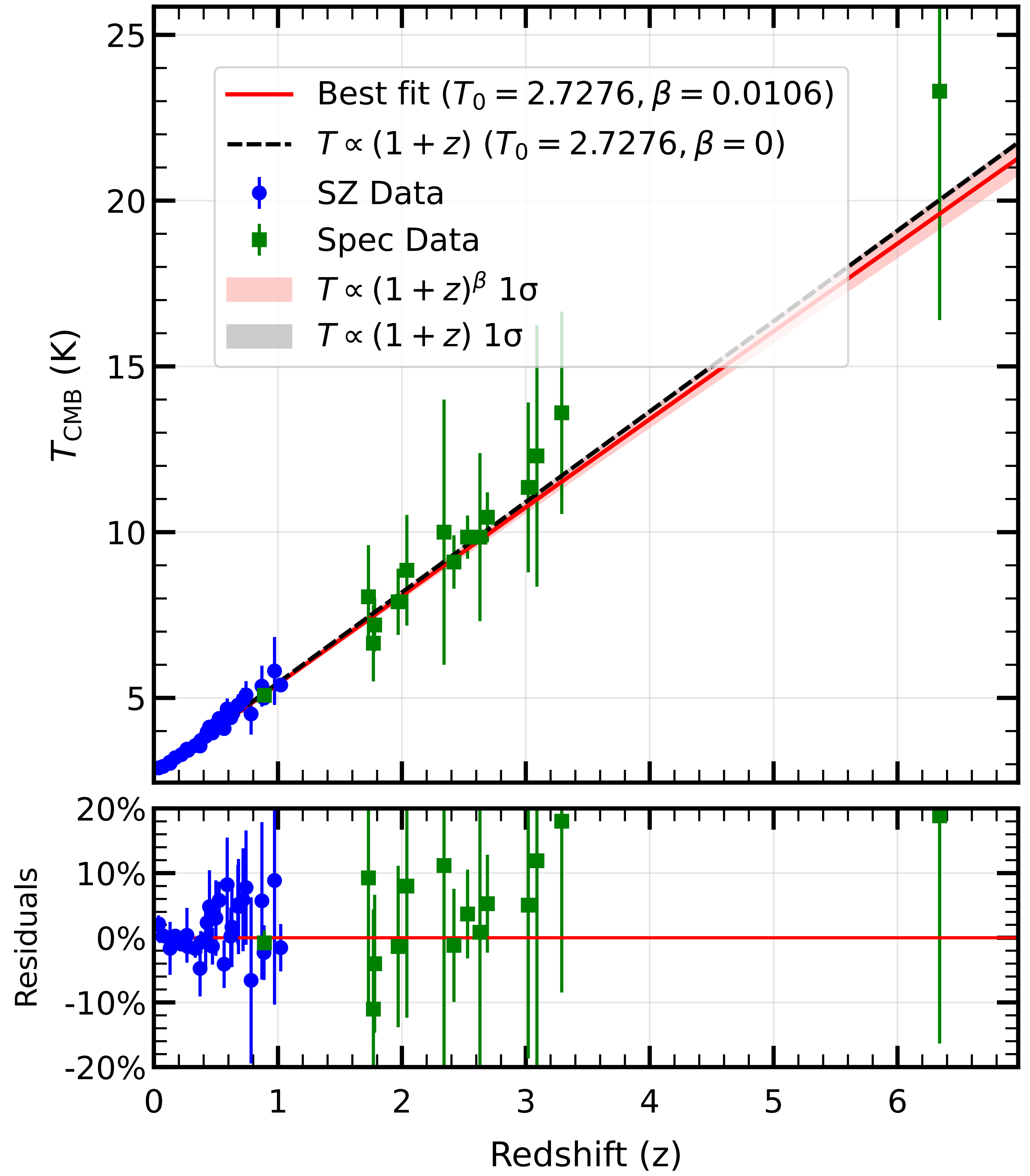
The DDR and the Temperature-Redshift relation

Based on: [Ruchika, W. Giarè, E. M. Teixeira, and A. Melchiorri: arxiv:2505.02909]

The DDR and the $T_{CMB}(z)$ relation

$$T_{\mathrm{CMB}}(z) = T_0(1+z)^{1-\beta} \;\Rightarrow\; D_L(z) = D_A(z)(1+z)^{2-\frac{3}{2}\beta}$$

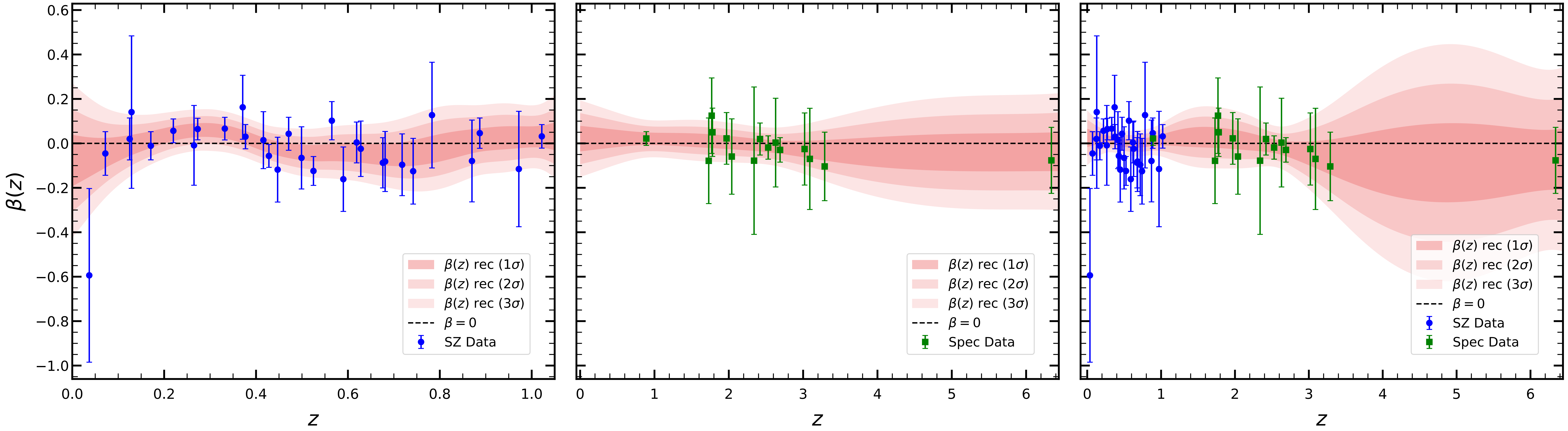
- Violations of the DDR associated with modifications to redshift evolution of the temperature of CMB photons
- Gaussian Process reconstruction and χ^2 minimisation of the parameter β using latest SZ effect measurements and molecular line excitation data
- Agreement with $\beta = 0$
- With $T(z)$ data we find $D_L(z)/D_A(z) \sim (1+z)^{2.0159 \pm 0.0186}$
- In DDR violation we found $D_L(z)/D_A(z) \sim (1+z)^{1.866 \pm 0.021}$
- Hint for SN/low-z systematics or modified background (e.g. DDE) expressed as DDR violation



[Ruchika, W Giarè, EMT, A. Melchiorri.: arXiv: 2505.02909]

The DDR and the $T_{CMB}(z)$ relation

$$T_{\rm CMB}(z) = T_0(1+z)^{1-\beta} \Rightarrow \beta(z) = 1 - \frac{\ln\left(T_{\rm CMB}(z)/T_0\right)}{\ln(1+z)}$$



[Ruchika, W Giarè, EMT, A. Melchiorri.: arXiv: 2505.02909]

Conclusions

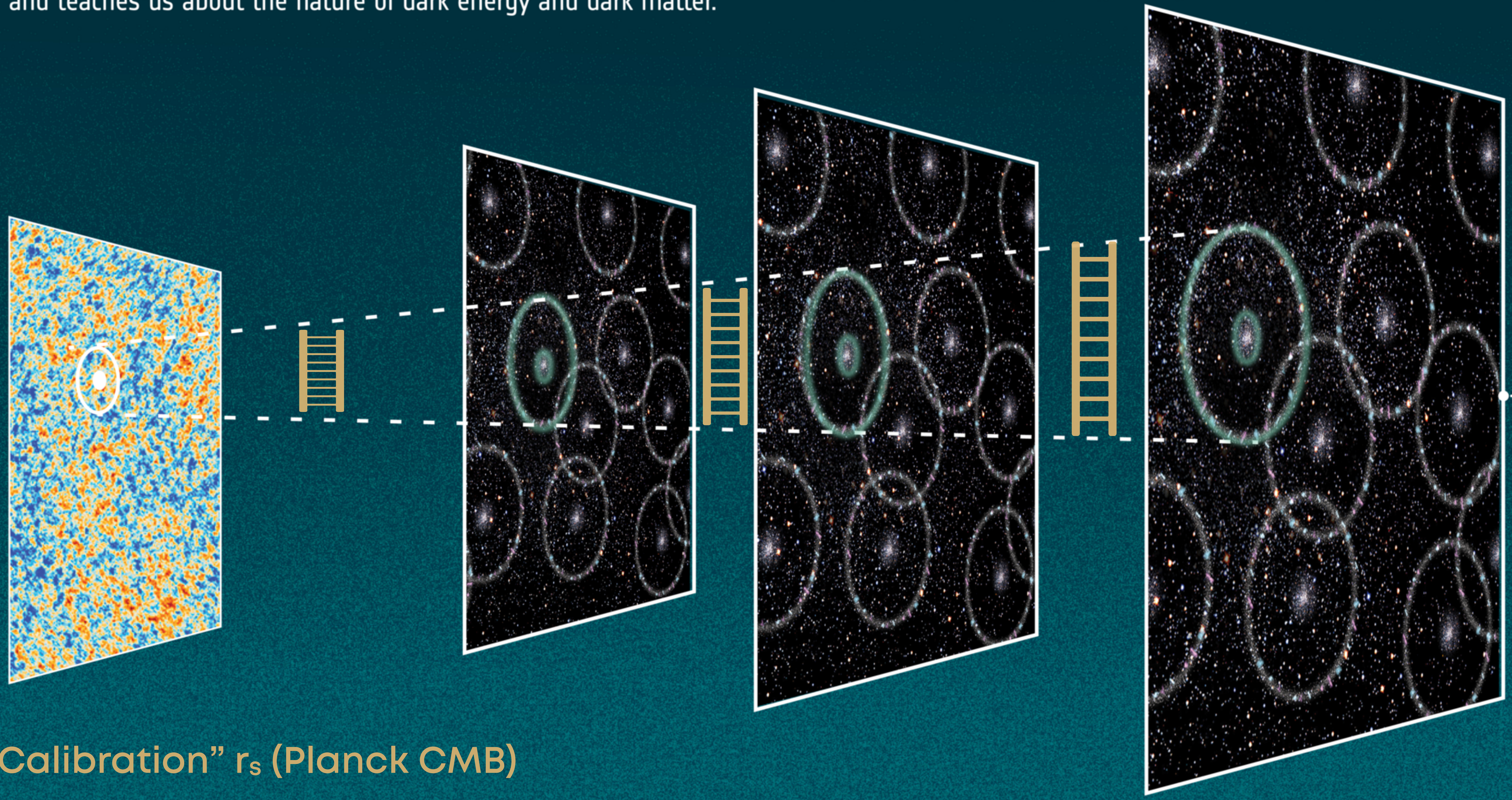
- The H_0 tension can be recast as a tension in distances
- Resolution of the tension with a preference for a constant shift in the calibration of the SN and BAO distances
- All parametrisations are preferred over ΛCDM, although not all can resolve the tension with SH0ES
- The data currently favours two possibilities:
  1. Constant violation of the DDR (equivalent to a calibration shift), $D_L(z)/D_A(z) \simeq 0.925(1+z)^2$
  2. Change in the power-law redshift-dependence of the DDR, restricted to z ≲ 1, $D_L(z)/D_A(z) \simeq (1+z)^{1.866}$, together with a phantom dark energy equation of state $w \sim -1.155$
- Disentangle DDR-violation models and 'early-universe' models with future independent and precise measurements of H_0

Thank you for your attention!

Illustration Credits: Inês Viegas Oliveira (ivoliveira.com)

WHAT EUCLID WILL MEASURE: BARYONIC ACOUSTIC OSCILLATIONS

When the early Universe first expanded, the formation of protons and neutrons created sound waves (bubbles) that rippled through the hot particle-radiation soup. About 300 000 years after the Big Bang, when the Universe had cooled down enough for atoms to form and light to travel freely, these waves froze in place. Over time, slightly more galaxies formed in clusters along the frozen ripples. The ripples stretched as the Universe expanded, increasing the distance between galaxies. Euclid will study the distribution of galaxies over immense distances, teasing out these ripple patterns and determining their size. This enables us to measure accurately the accelerated expansion of the Universe and teaches us about the nature of dark energy and dark matter.



Artist's impression of the pattern of baryonic acoustic oscillations imprinted on the large-scale distribution of galaxies (exaggerated)

"Calibration" r_s (Planck CMB)

Distances (DESI BAO)

Source: ESA and the Planck Collaboration / Gabriela Secara / Perimeter Institute

D_A from BAO

Distance Duality Relation (DDR)

$$D_L(z) = (1+z)^2 D_A(z)$$

[I. M. H. Etherington (1933)]

D_L from SN1a

$$ds^2 = -\,dt^2 + a^2(t)dr^2 + r^2 d\Omega^2$$

Photons on null geodesics

$$D_M = r(z) = \int_0^z \frac{dz}{H(z)}$$

$$D_A(z) = \frac{\Delta x}{\Delta \theta}$$

$$D_A(z) = \frac{D_M(z)}{(1+z)}$$

$\Delta x = a(t) D_M \Delta\theta$ with $\Delta\theta \ll 1$

$$\theta_{\mathrm{BAO}} = \frac{r_s}{(1+z)D_A(z)} \text{ with } r_s = \int_{z_d}^{0} \frac{c_s}{H} dz$$

$$D_L^2(z) \equiv \frac{L}{4\pi F}$$

$$D_L^2(z) = (1+z)^2 D_M^2(z)$$

$S = 4\pi a_0^2 D_M^2$, $\quad \dfrac{L_i}{L_f} = \dfrac{\Delta E_i}{\Delta E_F}\dfrac{\Delta t_f}{\Delta t_i} = (1+z)^2$

$$\mu(z) = 5\log_{10}\left[\frac{D_L(z)}{10\,\mathrm{pc}}\right] \text{ and } \mu_{\mathrm{SN}}(z) = m(z) - M_b$$

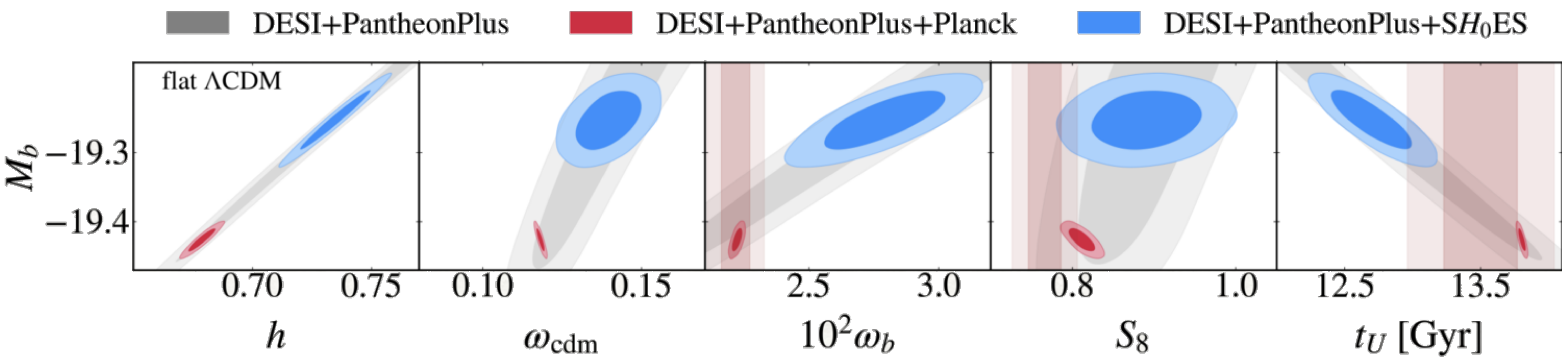
NEW PARALLAX LIMIT
Parallax of Cepheids in the Milky Way
OLD PARALLAX LIMIT
Earth, June
Sun
Earth, December
Galaxies hosting Cepheids and Type Ia supernovae
Light redshifted (stretched) by expansion of space
Distant galaxies in the expanding universe hosting Type Ia supernovae
Calibration M_b (SH0ES)
Distances (Pantheon+)
0 – 10 K LY
10 Thousand – 100 Million Light-years
100 Million – 1 Billion Light-years
[CREDIT: NASA/ESA/HUBBLE]

Breaking the DDR

- Assuming ΛCDM the SH0ES calibration intruduces several inconsistencies:

1. H_0 tension: unreconciable with CMB
2. BBN: larger wb to adjust lower rs
3. S_8: increases because of larger wm
4. Age of the Universe: younger by about 1 Gyr

- Becomes a challenge to put out all the fires



[Poulin et al.: arXiv: 2407.18292]

Evidence for constant DDR

$$D_L(z) = (1+z)^2 D_A(z)\,(1+\alpha_0)$$

ΛCDM + $\eta(z)$ for *Planck* 2018 + DESI + PantheonPlus + SH_0ES prior

Parameter	**ΛCDM**	**M1**	**M2**	**M3**	**M1 ($z_* = 0.9$)**	**M3 ($z_* = 0.9$)**
α_0	$--$	-0.075 ± 0.012	-0.070 ± 0.013	-0.049 ± 0.015	-0.076 ± 0.012	-0.066 ± 0.017
α_1	$--$	$--$	-0.014 ± 0.010	$--$	-0.039 ± 0.024	0.010 ± 0.026
M_B	-19.395 ± 0.011	-19.254 ± 0.026	-19.253 ± 0.026	-19.370 ± 0.013	-19.253 ± 0.027	-19.367 ± 0.013
r_s	147.62 ± 0.22	147.35 ± 0.23	147.42 ± 0.23	147.73 ± 0.22	147.32 ± 0.23	147.72 ± 0.22
H_0	68.89 ± 0.38	68.00 ± 0.41	68.18 ± 0.42	69.10 ± 0.38	67.93 ± 0.41	69.08 ± 0.38
Ω_m	0.2953 ± 0.0048	0.3066 ± 0.0054	0.3042 ± 0.0055	0.2925 ± 0.0047	0.3076 ± 0.0055	0.2928 ± 0.0047
$\Delta\chi^2_{\rm min}$	$--$	-32.84	-34.73	-10.27	-35.40	-17.25
$\log \mathcal{Z}_M/\mathcal{Z}_{\Lambda\rm CDM}$	0	13.6	10.9	2.3	12.3	3.3

TABLE II: Observational constraints at a 68% confidence level on the cosmological parameters for a ΛCDM cosmology with different models of DDR violation, inferred from analyses of the combination of *Planck* 2018 data, DESI BAO and PantheonPlus SNIa calibrated with a SH_0ES prior.

Evidence for constant DDR

$$D_L(z) = (1+z)^2 D_A(z)\,(1+\alpha_0)$$

ΛCDM + $\eta(z)$ for *Planck* 2018 + DESI + PantheonPlus						
Parameter	**ΛCDM**	**M1**	**M2**	**M3**	**M1** ($z_* = 0.9$)	**M3** ($z_* = 0.9$)
α_0	$--$	$-0.088^{+0.096}_{-0.11}$	$-0.062^{+0.12}_{-0.090}$	-0.024 ± 0.016	$-0.066^{+0.12}_{-0.086}$	-0.040 ± 0.017
α_1	$--$	$--$	-0.014 ± 0.010	$--$	$-0.027^{+0.12}_{-0.094}$	0.026 ± 0.027
M_B	-19.422 ± 0.012	-19.21 ± 0.23	$-19.26^{+0.19}_{-0.29}$	-19.407 ± 0.015	$-19.27^{+0.19}_{-0.29}$	-19.402 ± 0.015
r_s	147.36 ± 0.23	147.35 ± 0.22	147.43 ± 0.23	147.44 ± 0.23	147.32 ± 0.23	147.44 ± 0.23
H_0	68.00 ± 0.40	68.00 ± 0.39	68.17 ± 0.41	68.20 ± 0.42	67.91 ± 0.41	68.22 ± 0.42
Ω_m	0.3065 ± 0.0054	0.3065 ± 0.0052	0.3043 ± 0.0054	0.3039 ± 0.0055	0.3078 ± 0.0054	0.3036 ± 0.0055
$\Delta\chi^2_{\rm min}$	$--$	-0.33	-2.44	-2.51	-3.34	-7.36
$\log \mathcal{Z}_M/\mathcal{Z}_{\Lambda\rm CDM}$	0	-0.6	-3.1	-2.1	-2	-2.1
GT w/ SH_0ES M_B	5.7σ	0.2σ	0.03σ	5.0σ	0.08σ	4.8σ

TABLE III: Same as Table II without the SH_0ES M_B prior. We also report the Gaussian tension (GT) with the SH_0ES M_B measurement defined in (18).

Evidence for dynamical DE

$$D_L(z) = (1+z)^2 D_A(z)\,(1+\alpha_0)$$

$w_0w_a + \eta(z)$ for *Planck* 2018 + DESI + PantheonPlus + SH$_0$ES prior						
Parameter	w_0w_a	**M1**	**M2**	**M3**	**M1** $(z_*=0.9)$	**M3** $(z_*=0.9)$
α_0	$--$	-0.066 ± 0.014	-0.060 ± 0.021	-0.090 ± 0.023	-0.071 ± 0.017	-0.134 ± 0.028 $[-0.128\pm0.021]$
α_1	$--$	$--$	-0.008 ± 0.021	$--$	-0.035 ± 0.029	-0.040 ± 0.029 $[--]$
w_0	-0.784 ± 0.067	-0.820 ± 0.065	-0.846 ± 0.091	-1.029 ± 0.089	-0.821 ± 0.075	-1.155 ± 0.095 $[-1.158\pm0.033]$
w_a	$-1.20^{+0.34}_{-0.30}$	$-0.78^{+0.32}_{-0.27}$	$-0.74^{+0.33}_{-0.29}$	$-0.52^{+0.36}_{-0.32}$	$-0.72^{+0.36}_{-0.30}$	$-0.09^{+0.37}_{-0.32}$ $[--]$
M_B	-19.350 ± 0.016	-19.254 ± 0.027	-19.254 ± 0.028	-19.283 ± 0.024	-19.253 ± 0.032	-19.264 ± 0.024
r_s	147.06 ± 0.25	147.13 ± 0.26	147.12 ± 0.26	147.06 ± 0.26	147.15 ± 0.31	147.11 ± 0.26
H_0	69.77 ± 0.60	68.00 ± 0.71	68.5 ± 1.3	72.08 ± 0.88	67.70 ± 0.89	72.77 ± 0.91
Ω_m	0.2939 ± 0.0055	0.3089 ± 0.0067	$0.305^{+0.011}_{-0.013}$	0.2756 ± 0.0070	0.3116 ± 0.0085	0.2700 ± 0.0070
$\Delta\chi^2_{\rm min}$	-20.37	-40.89	-40.16	-34.82	-42.75	-43.20 $[-41.86]$
$\log \mathcal{Z}_M/\mathcal{Z}_{\Lambda\rm CDM}$	5.6	12.7	10.4	10.5	11.5	12.4 [15.4]

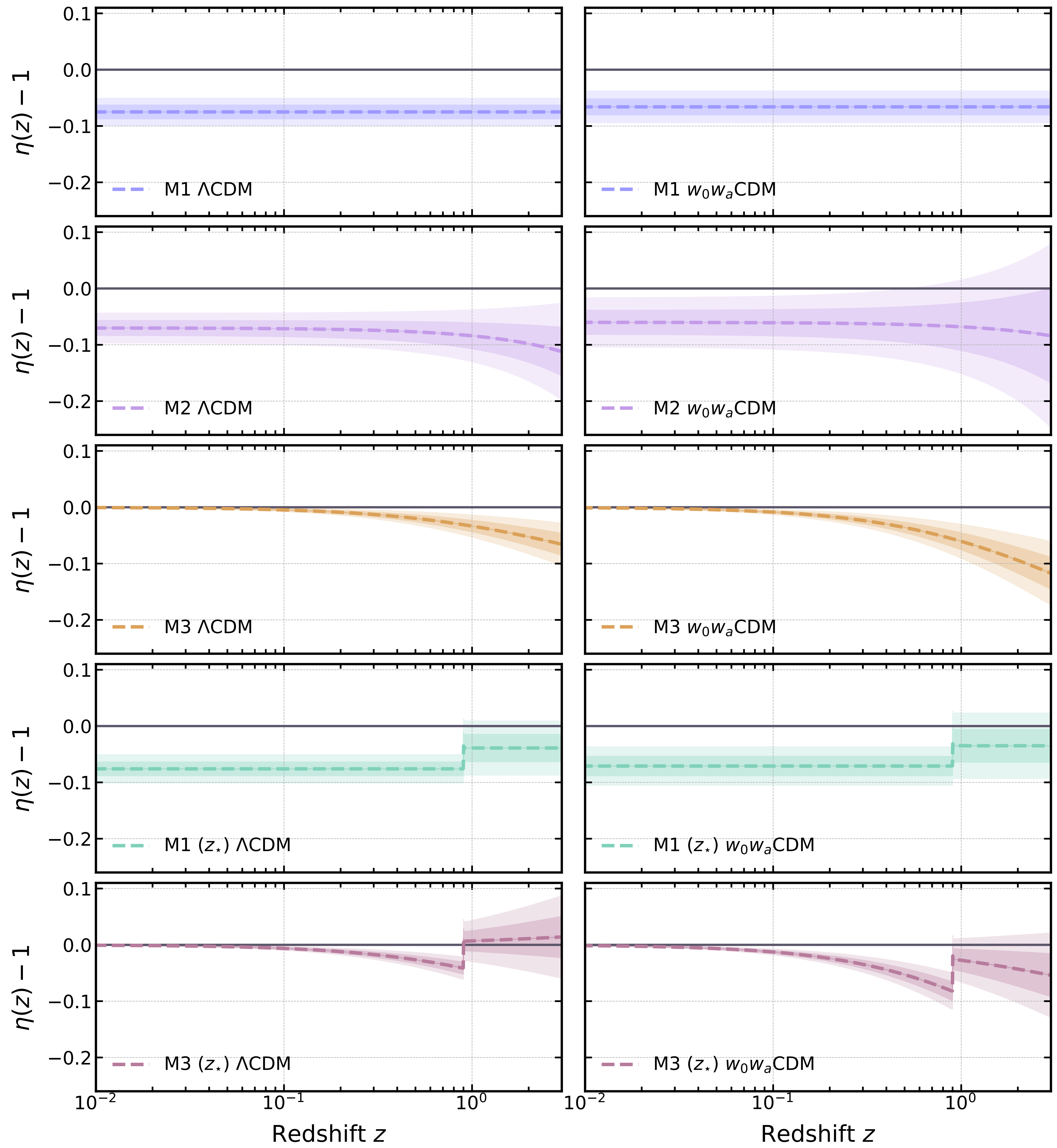
TABLE IV: Same as Table II, now in the w_0w_aCDM cosmology and with the SH$_0$ES M_B prior. In square brackets, we list the values for M3 $(z_*=0.9)$ with $\alpha_1 = w_a = 0$ (the corresponding cosmological parameters are consistent with the full case).

Evidence for dynamical DE

$$D_L(z) = (1+z)^2 D_A(z)\,(1+\alpha_0)$$

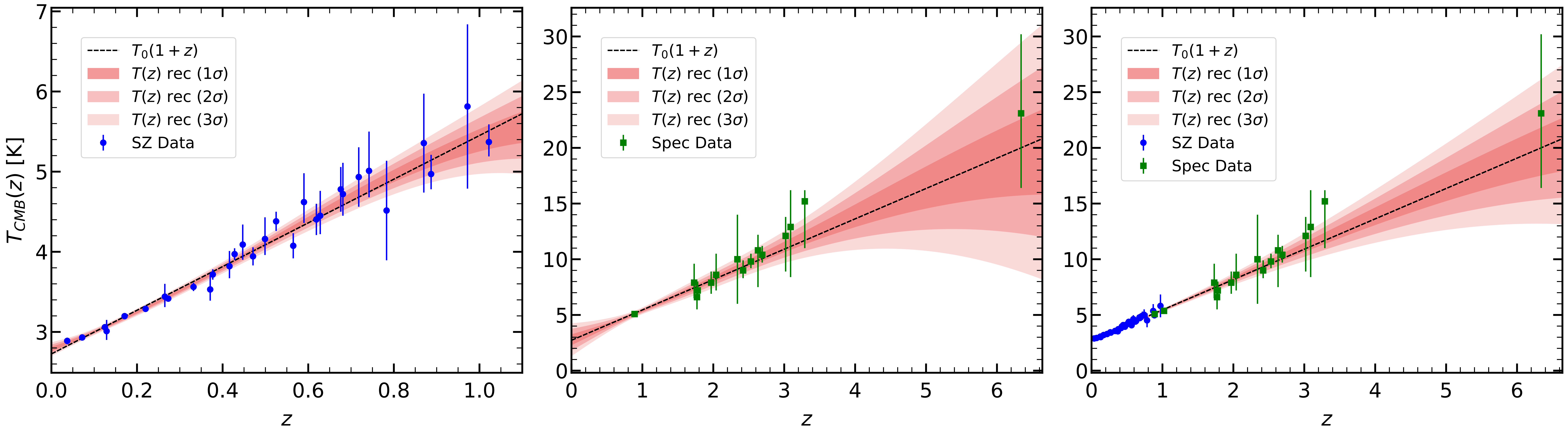
$w_0 w_a + \eta(z)$ for *Planck* 2018 + DESI + PantheonPlus						
Parameter	$w_0 w_a$	**M1**	**M2**	**M3**	**M1** $(z_* = 0.9)$	**M3** $(z_* = 0.9)$
α_0	$--$	-0.072 ± 0.096	$-0.057^{+0.12}_{-0.096}$	-0.012 ± 0.040	$-0.065^{+0.12}_{-0.093}$	-0.085 ± 0.054
α_1	$--$	$--$	-0.008 ± 0.021	$--$	$-0.03^{+0.12}_{-0.10}$	-0.011 ± 0.040
w_0	-0.822 ± 0.065	-0.821 ± 0.063	-0.846 ± 0.092	-0.85 ± 0.12	-0.825 ± 0.061	-1.04 ± 0.15
w_a	-0.77 ± 0.30	$-0.77^{+0.31}_{-0.27}$	$-0.74^{+0.33}_{-0.29}$	$-0.72^{+0.35}_{-0.32}$	$-0.69^{+0.29}_{-0.25}$	-0.28 ± 0.39
M_B	-19.404 ± 0.020	$-19.23^{+0.21}_{-0.27}$	$-19.25^{+0.20}_{-0.29}$	-19.390 ± 0.049	$-19.26^{+0.20}_{-0.29}$	-19.323 ± 0.060
r_s	147.13 ± 0.26	147.13 ± 0.26	147.12 ± 0.26	147.12 ± 0.26	147.16 ± 0.26	147.13 ± 0.26
H_0	67.99 ± 0.71	68.00 ± 0.70	68.4 ± 1.3	68.5 ± 1.7	67.65 ± 0.74	70.8 ± 2.1
Ω_m	0.3091 ± 0.0068	0.3090 ± 0.0067	$0.305^{+0.011}_{-0.013}$	0.305 ± 0.015	0.3118 ± 0.0071	0.286 ± 0.017
$\Delta\chi^2_{\rm min}$	-8.05	-7.95	-6.66	-8.16	-10.50	-11.06
$\log \mathcal{Z}_M/\mathcal{Z}_{\Lambda\rm CDM}$	-1.2	-1.4	-6.1	-2.7	-3	-2.8
GT w/ SH$_0$ES M_B	4.4σ	0.08σ	0.01σ	2.4σ	0.03σ	1.1σ

TABLE V: Same as Table IV without the SH$_0$ES M_B prior. We also report the Gaussian tension (GT) with the SH$_0$ES M_B measurement defined in (18).

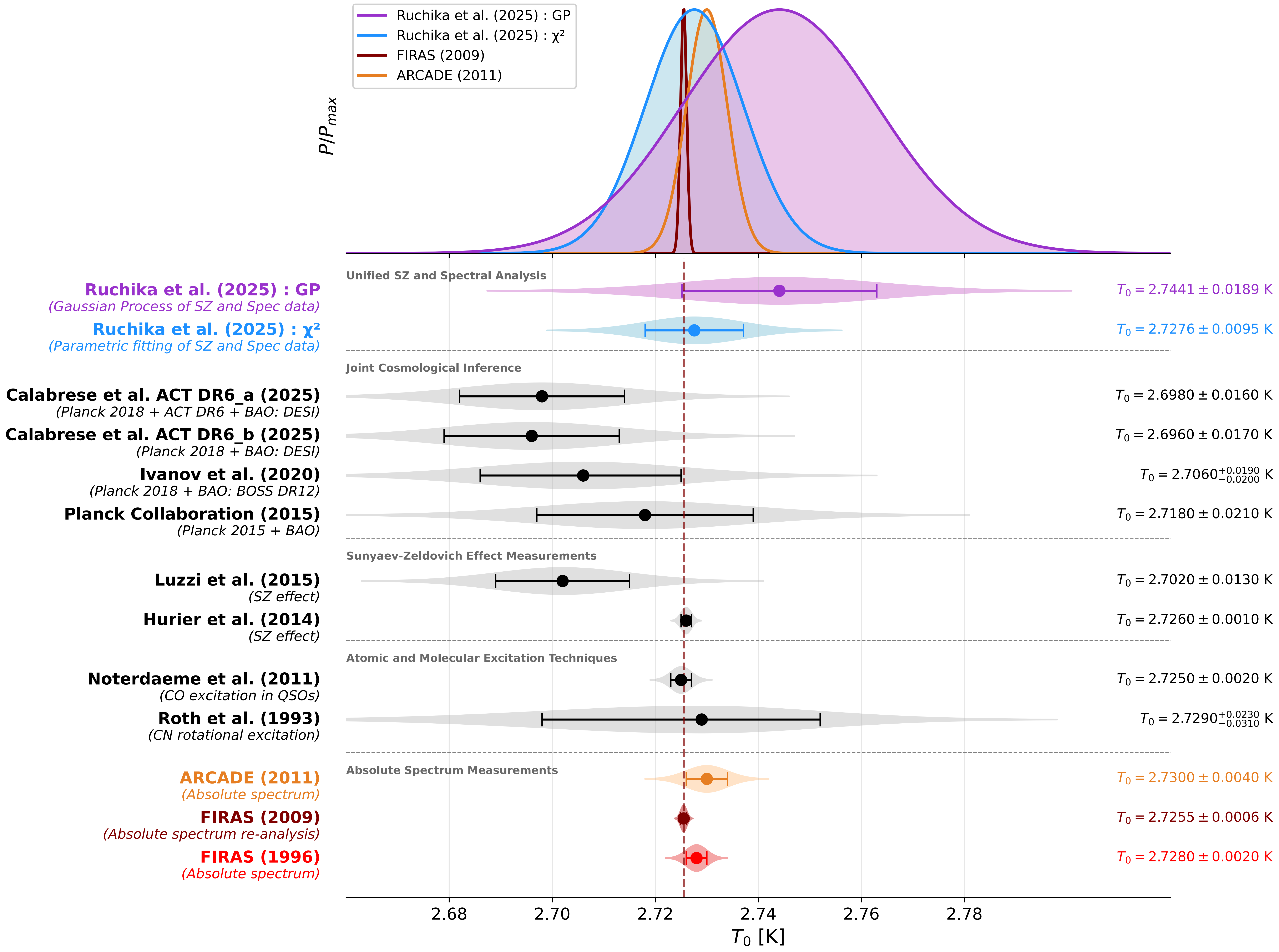

M1 ΛCDM
M1 w_0w_aCDM
M2 ΛCDM
M2 w_0w_aCDM
M3 ΛCDM
M3 w_0w_aCDM
M1 $(z_\star)$ ΛCDM
M1 $(z_\star)$ w_0w_aCDM
M3 $(z_\star)$ ΛCDM
M3 $(z_\star)$ w_0w_aCDM
$\eta(z) - 1$
Redshift z
0.1
0.0
−0.1
−0.2
10^{-2}
10^{-1}
10^{0}

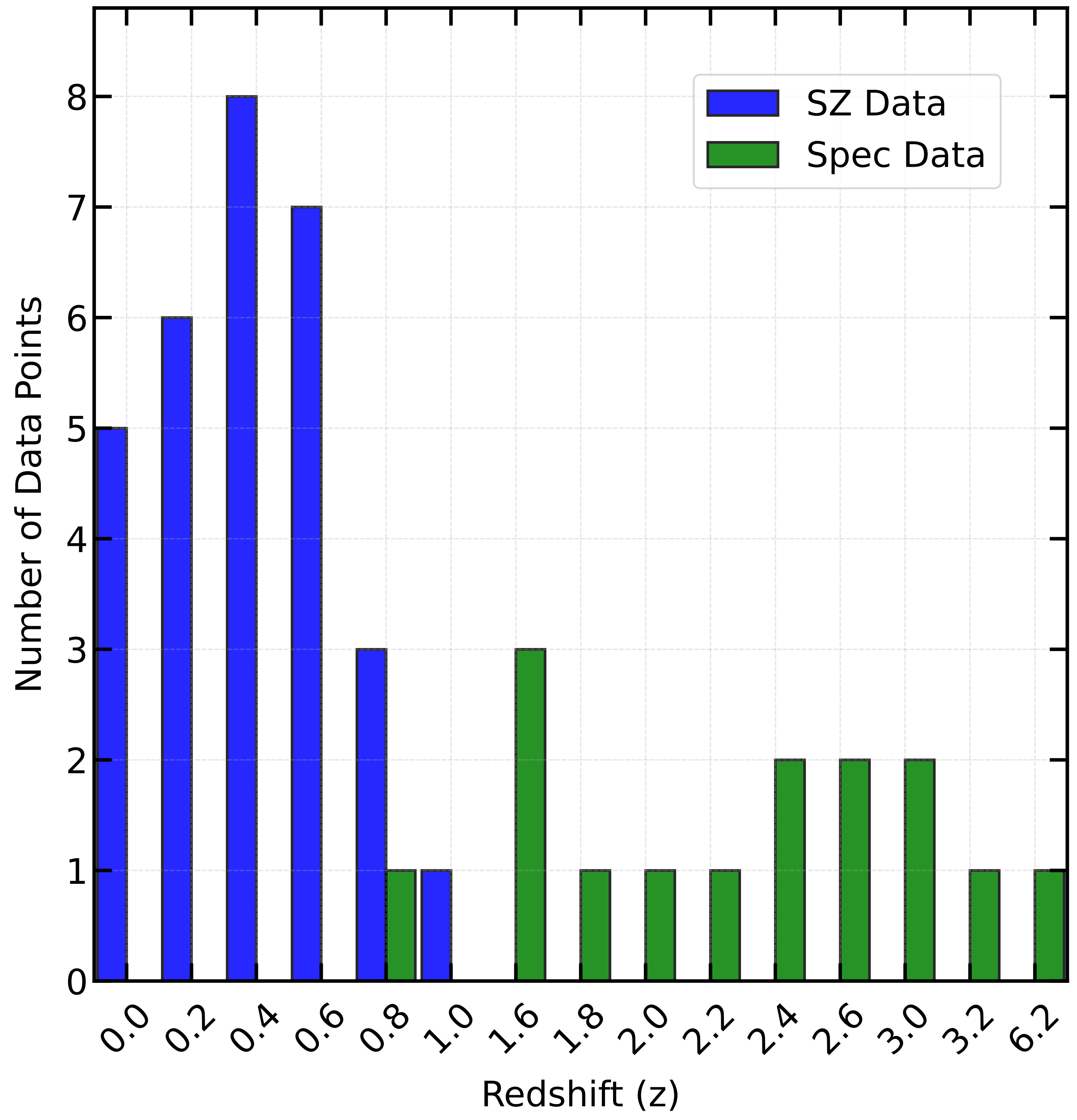
The DDR and the $T_{CMB}(z)$ relation

$$T_{\rm CMB}(z) = T_0(1+z) \;\Rightarrow\; T_0 = 2.744 \pm 0.019\,\mathrm{K}$$



[Ruchika, W Giarè, EMT, A. Melchiorri.: arXiv: 2505.02909]





[Ruchika et al.: arXiv: 2505.02909]