Modern Statistical Methods for Cosmological Inference: Samplers, Likelihoods, and Beyond

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Based on Universe 11 (2025) 68 (arXiv:2501.06022) and Physics of the Dark Universe (2025) 49 (arXiv: 2504.18416)

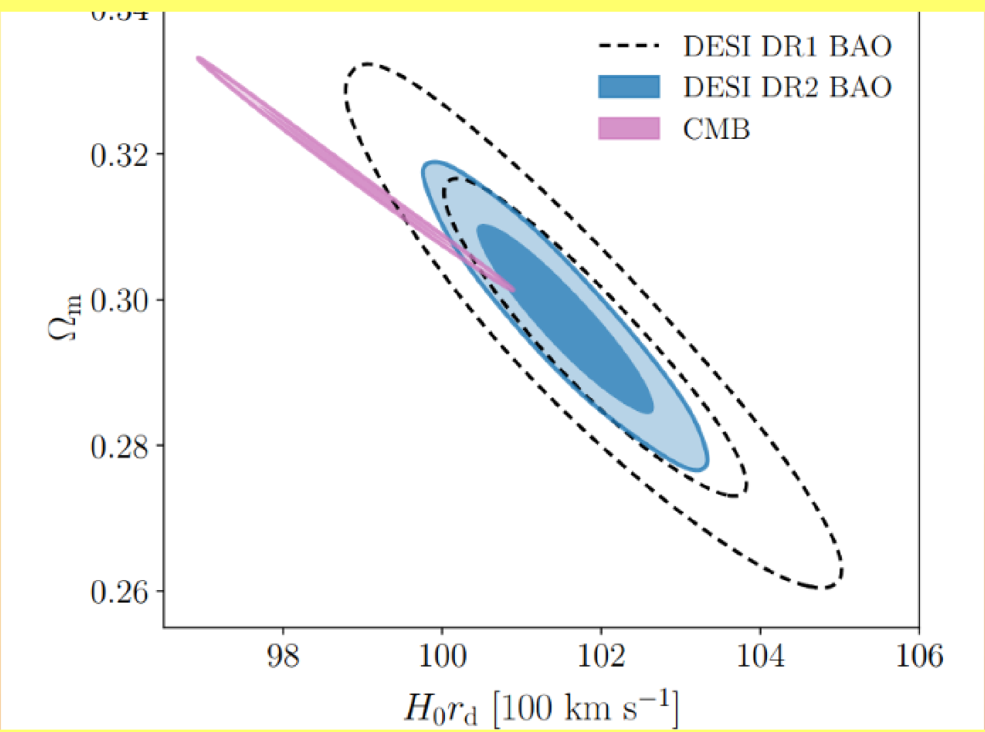
CosmoVerse, 3d Annual Conference, Istanbul, Turkiye, 24-26.06.2025

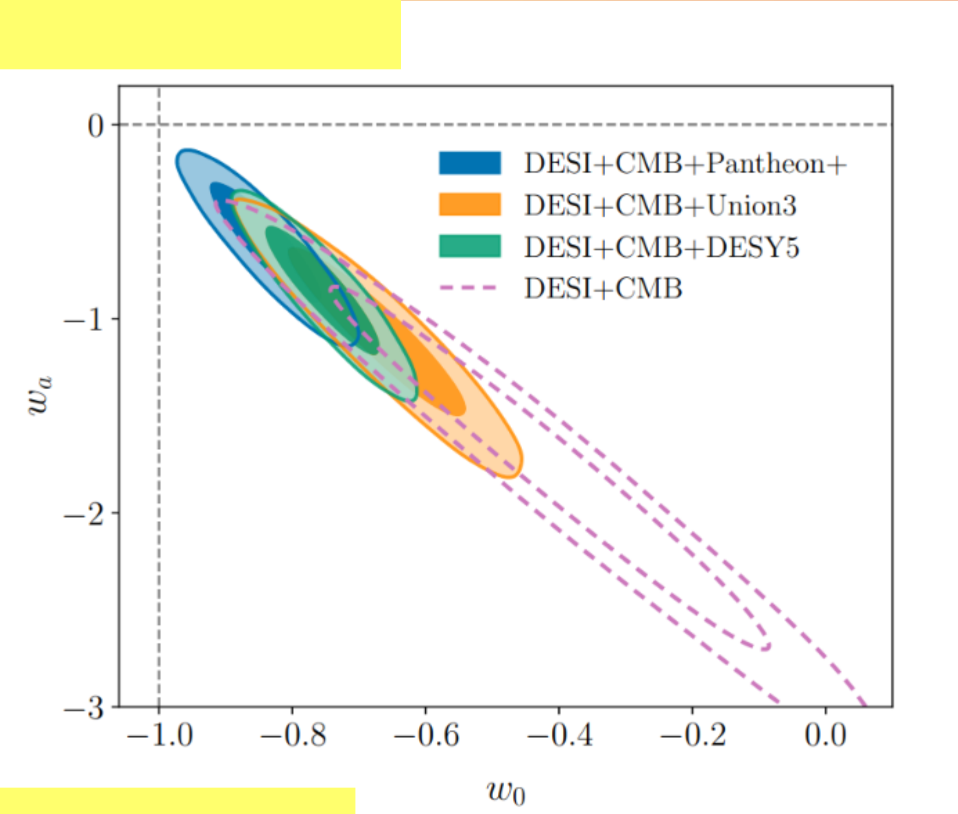
ИЯИЯЕ INRNE

БЪЛГАРСКА АКАДЕМИЯ на НАУКИТЕ 1869

The state of the Hubble tension

The Hubble tensions is at 5.3σ as of 2023! 4.5σ from DESI+BBN
The novelty: 1.7σ-3.3σ evidences for wwaCDM!





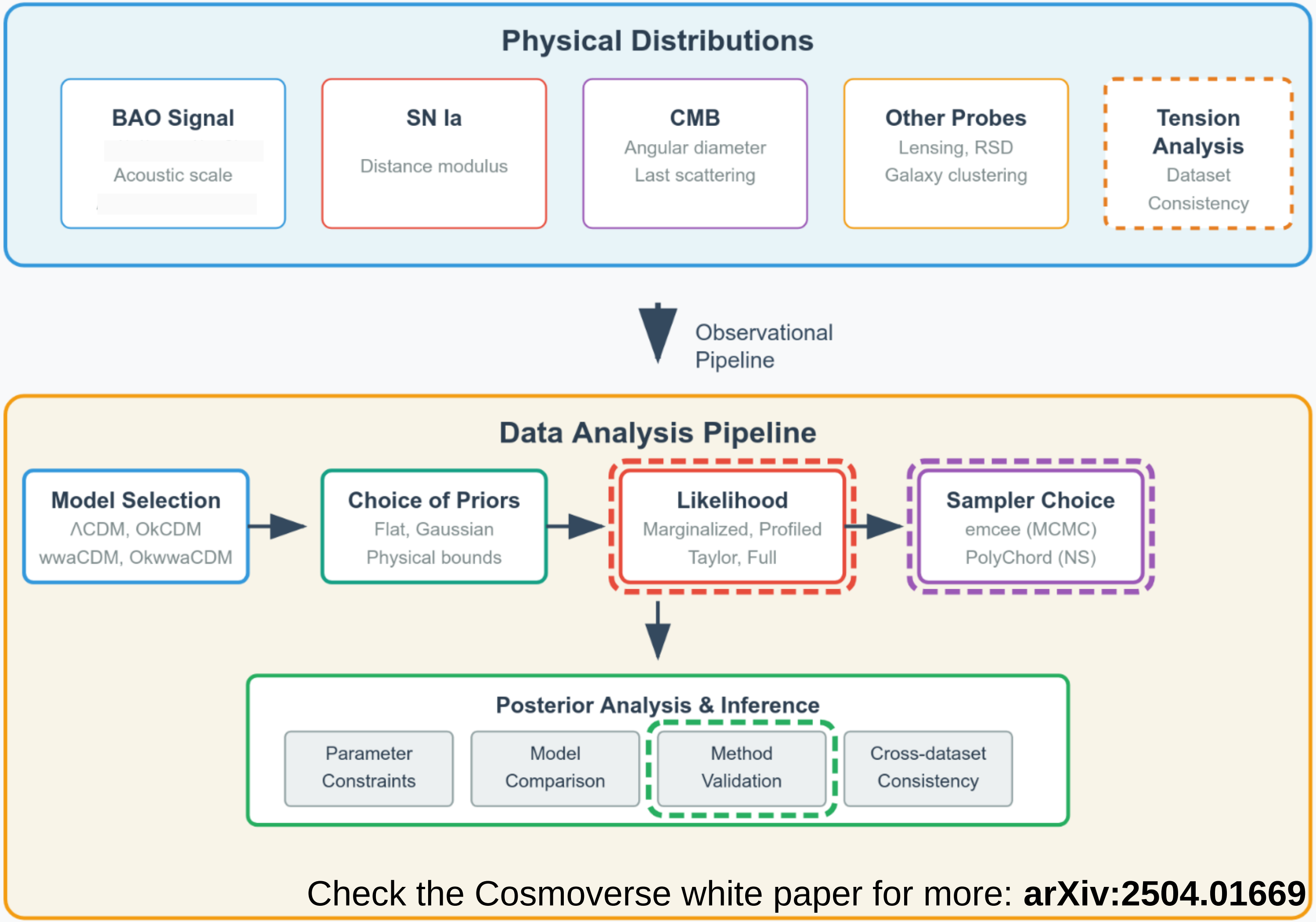
Obviously the situation got even more interesting.
But how trustable are the results statistically?

Aghanim et al. , Astron. Astrophys. 641, 2020

DESI collab., 2503.14738

Riess, A. G. et al. ApJL 934 (2022)

From Physical Reality to Statistical Inference



Our test problems:

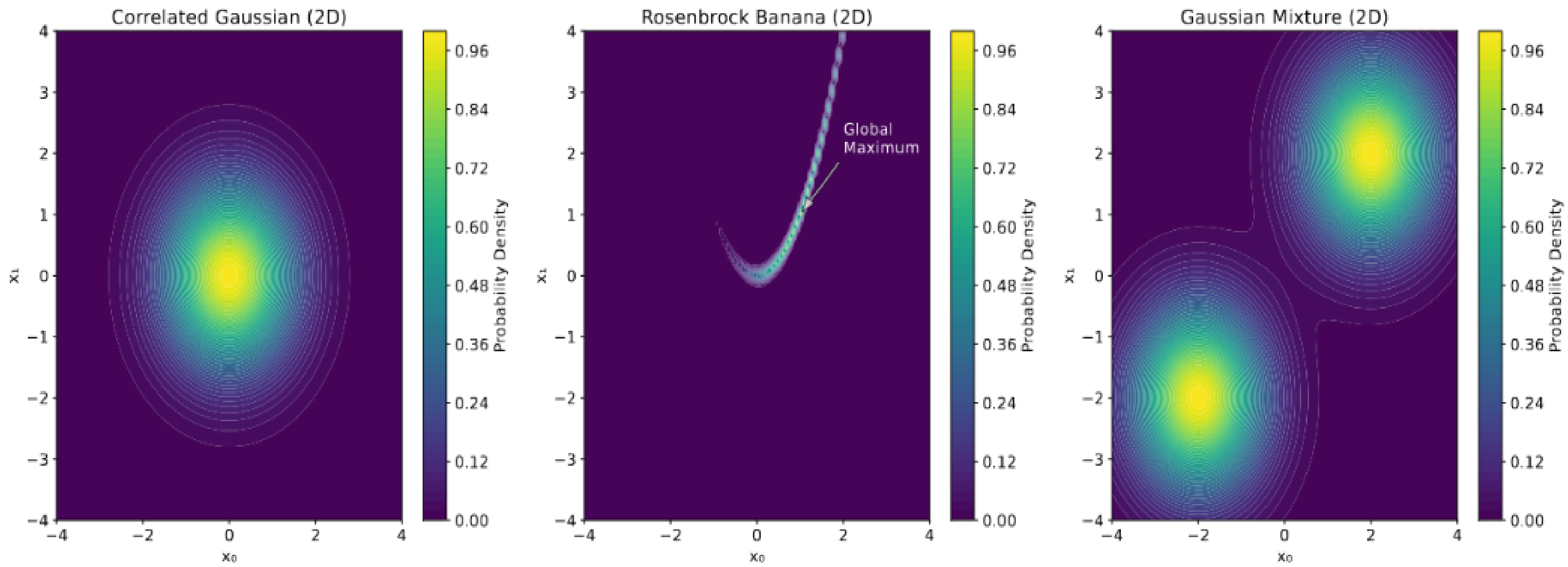
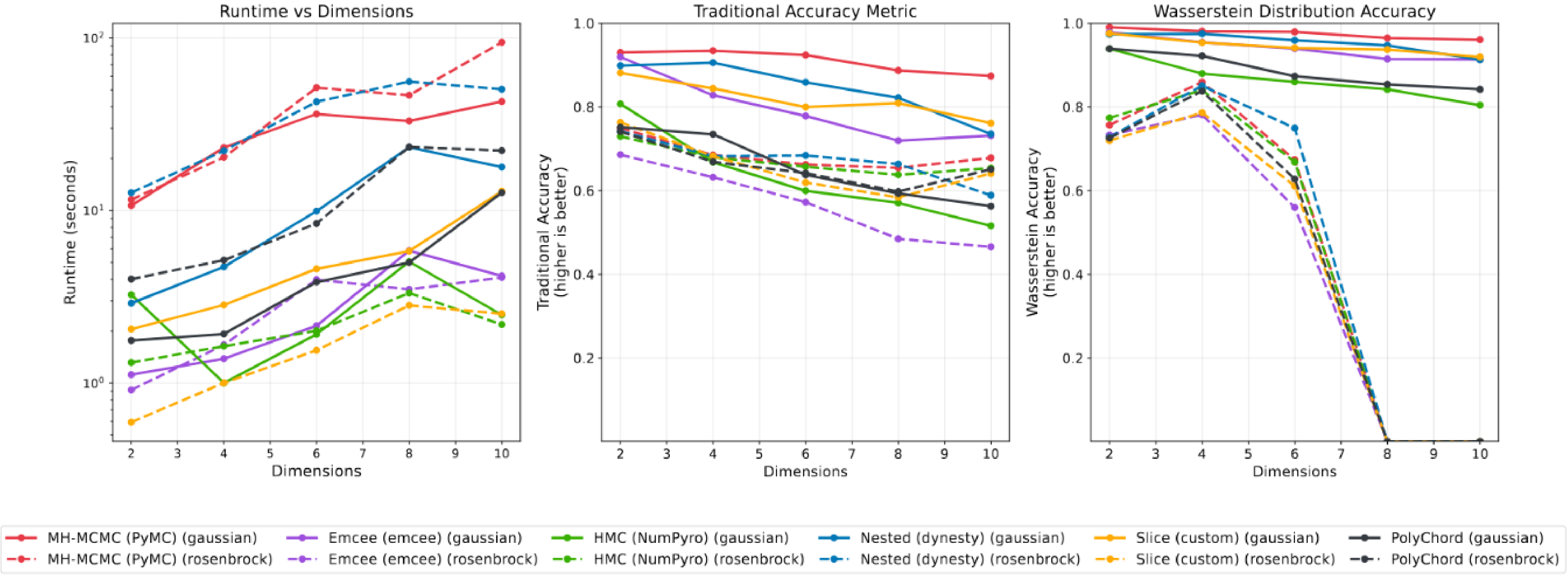


Figure 1. The surface plots corresponding to the three test problems. The global maximum that the sampler needs to find is marked in the case of the Rosenbrock distribution; the two others correspond to single and double Gaussians, respectively.

D. Staicova, 2501.06022

How the different samplers perform:



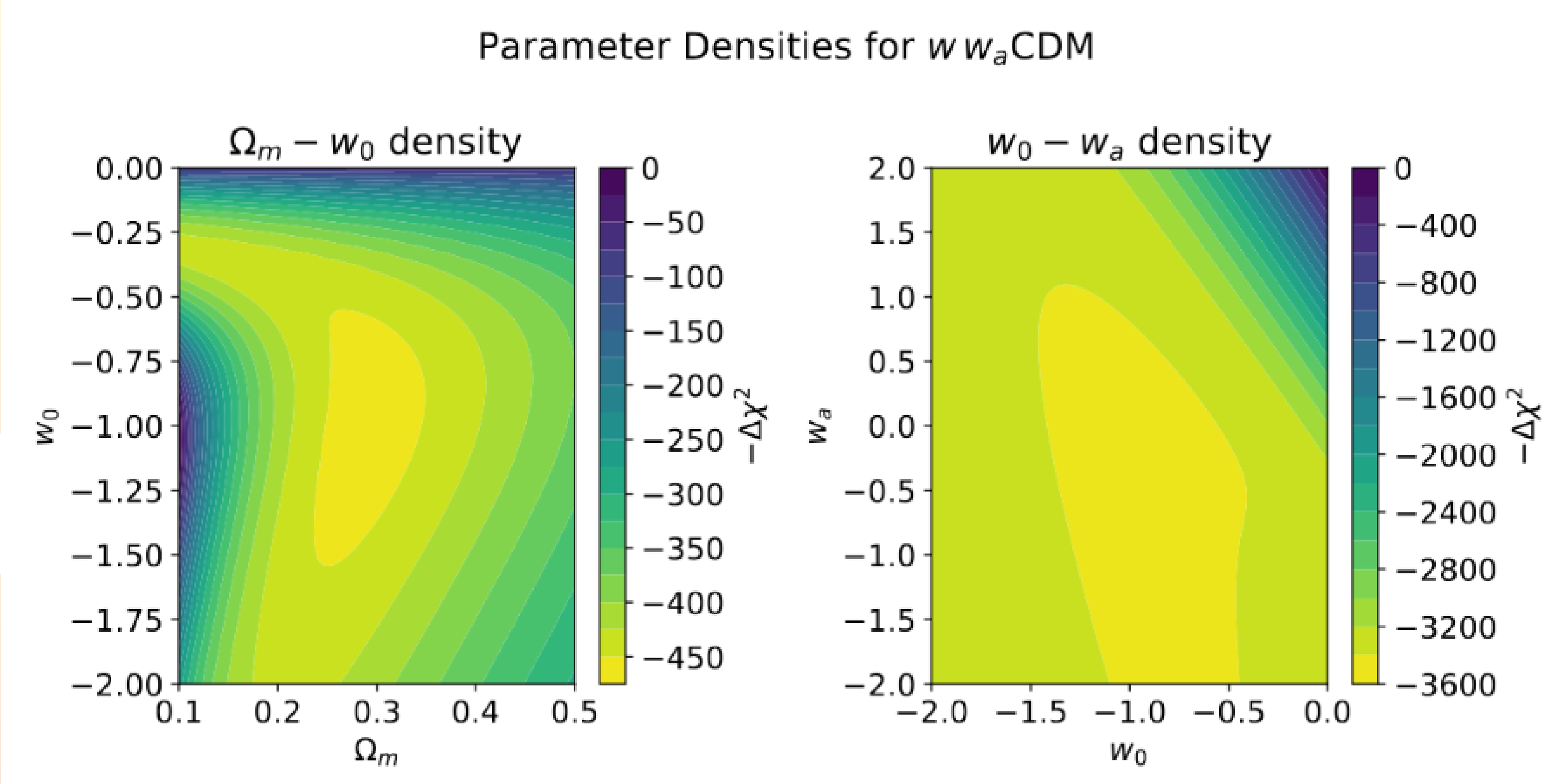
CosmoVerse White paper, 2504.01669

In BAO studies...

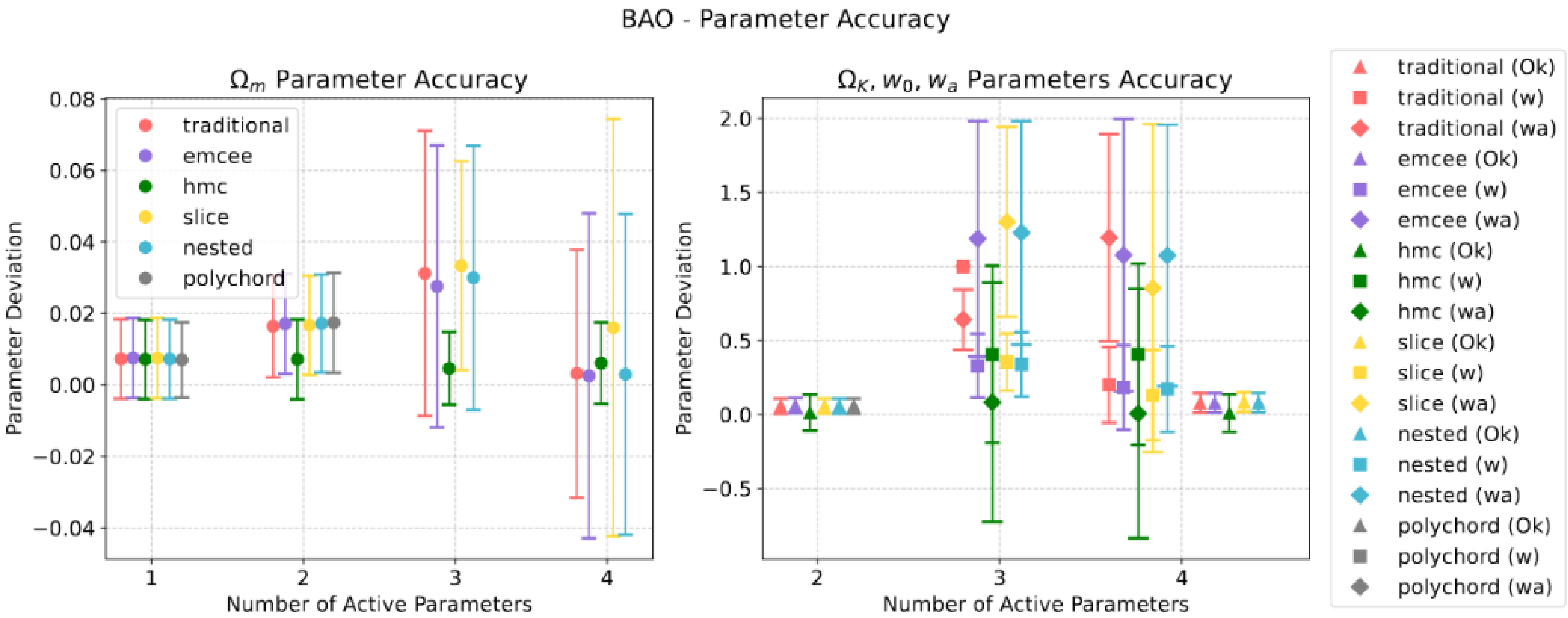
3d BAO

$$\frac{D_H}{r_d} = \frac{c}{H_0 r_d} \frac{1}{E(z)},$$

$$\frac{D_A(z)}{r_d} = \frac{c}{r_d H_0} \int_0^z \frac{dz'}{E(z')}$$



In terms of cosmological parameters



Let's also examine the likelihoods

- Full likelihood

$$\chi^2_{\rm full}(\boldsymbol{\theta}, H_0, r_d) = \sum_{i,j} [d_i - t_i(\boldsymbol{\theta}, H_0, r_d)] C^{-1}_{ij} [d_j - t_j(\boldsymbol{\theta}, H_0, r_d)]$$

$$\frac{D_M(z)}{r_d} = c\beta \cdot g(\boldsymbol{\theta}, z)$$

$$\frac{D_H(z)}{r_d} = c\beta \cdot f(\boldsymbol{\theta}, z)$$

$$\frac{D_V(z)}{r_d} = c\beta \cdot [z \cdot g(\boldsymbol{\theta}, z)^2 \cdot f(\boldsymbol{\theta}, z)]^{1/3}$$

where $g(\boldsymbol{\theta}, z) = S_k(\chi(z))$ and $f(\boldsymbol{\theta}, z) = 1/E(z)$

- The problem – BAO data does not constrain H_0 and rd but their combination $\beta = 1/ H_0 r_d$
- Either calibrate with early uninverse (CMB) or late (SN)
- Or assume matter content for early universe (for r_d) - BBN
- The **choice of priors** leads to a tension

Other options for likelihoods

- Marginalized

$$\chi^2_{\mathrm{marg}}(\boldsymbol{\theta}) = C - \frac{B(\boldsymbol{\theta})^2}{4A(\boldsymbol{\theta})} + \ln\left(\frac{A(\boldsymbol{\theta})}{2\pi}\right)$$

$$A(\boldsymbol{\theta}) = \sum_{i,j} p_i(\boldsymbol{\theta}) C^{-1}_{ij} p_j(\boldsymbol{\theta})$$

$$B(\boldsymbol{\theta}) = \sum_{i,j} d_i C^{-1}_{ij} p_j(\boldsymbol{\theta}) + \sum_{i,j} p_i(\boldsymbol{\theta}) C^{-1}_{ij} d_j$$

$$C = \sum_{i,j} d_i C^{-1}_{ij} d_j$$

- Profile

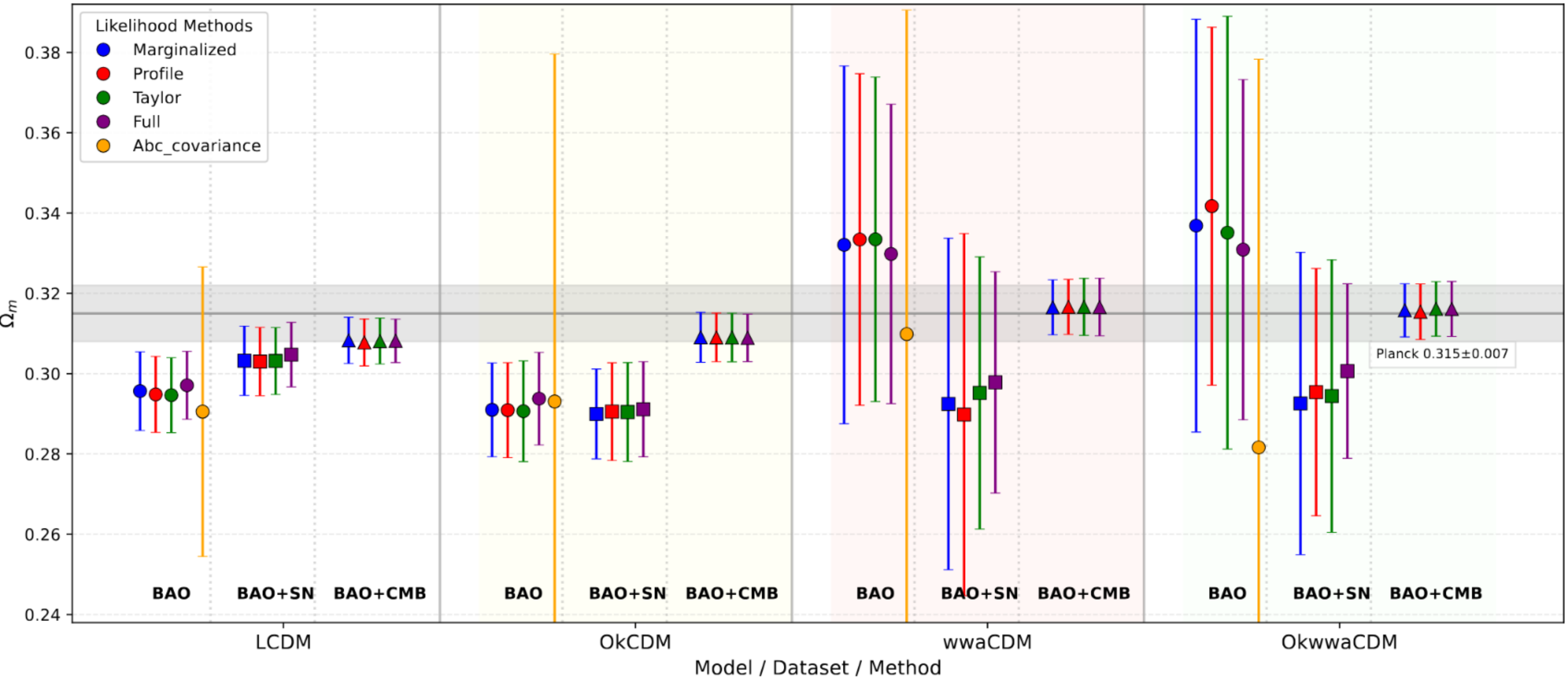
$$\chi^2_{\mathrm{prof}}(\boldsymbol{\theta}) = \min_{\beta} \chi^2(\boldsymbol{\theta}, \beta)$$

Taylor expansion

$$\chi^2_{\mathrm{taylor}}(\boldsymbol{\theta}) = C - \frac{B(\boldsymbol{\theta})^2}{A(\boldsymbol{\theta})} - \ln\left(\frac{2\pi}{A(\boldsymbol{\theta})}\right)$$

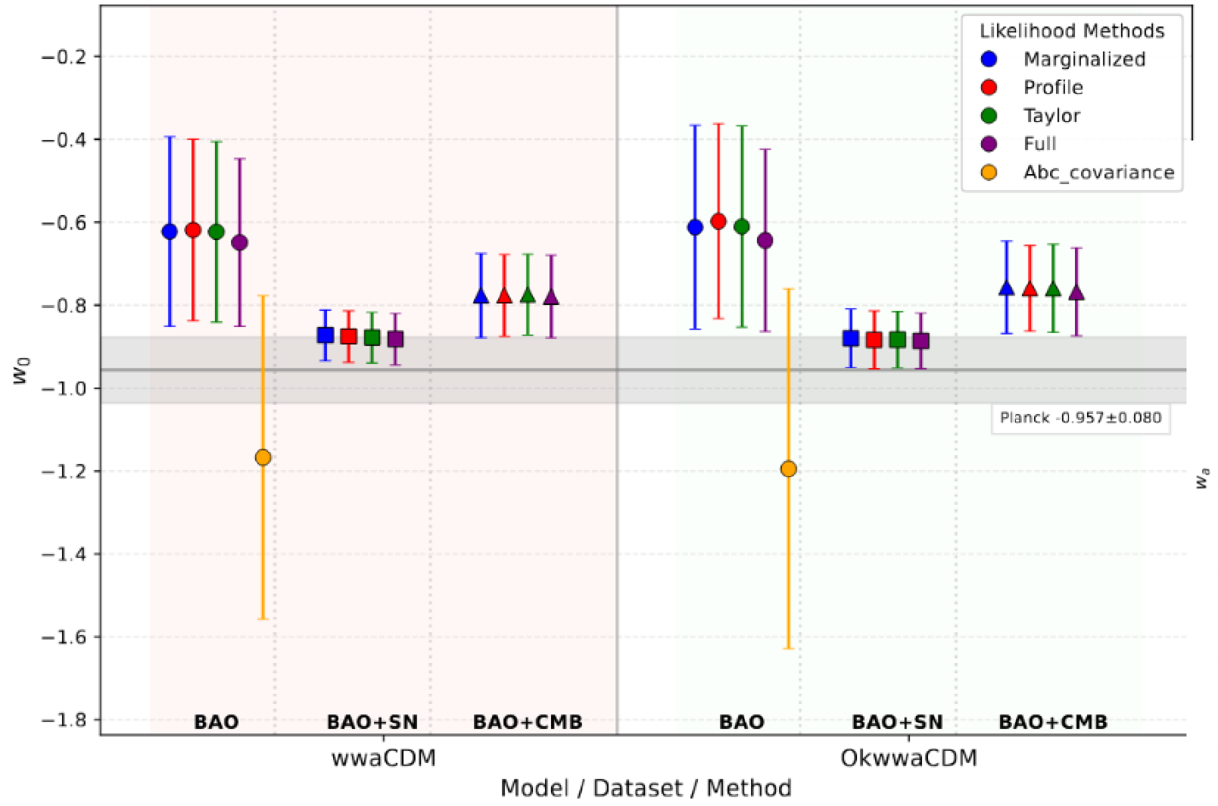
Marginalized: integrate β out, Taylor: expands around the optimal β (quadratic approx), i.e between Marginalized and Profiled

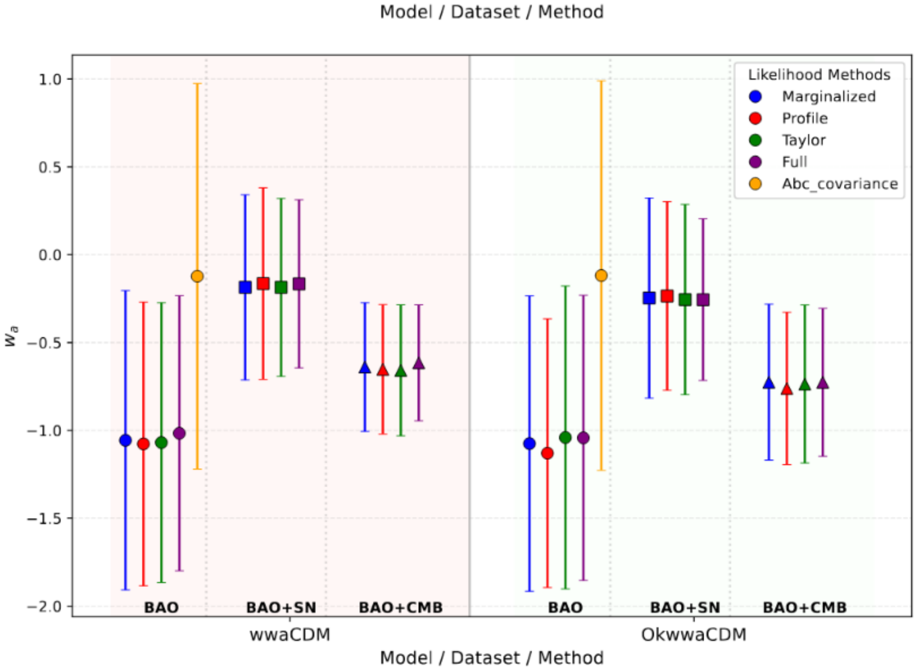
Consistency across likelihoods but not between models!

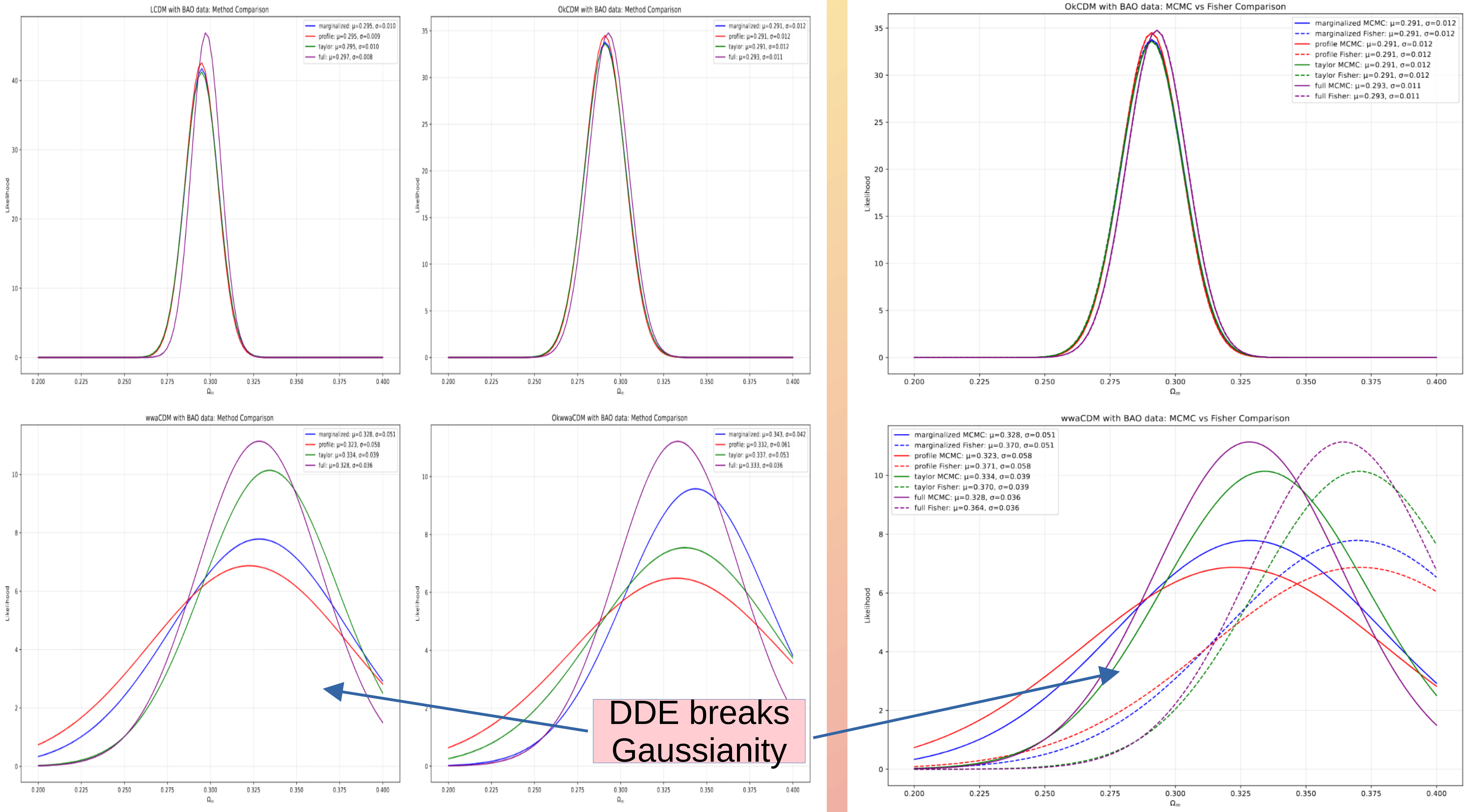


Dynamical Dark Energy

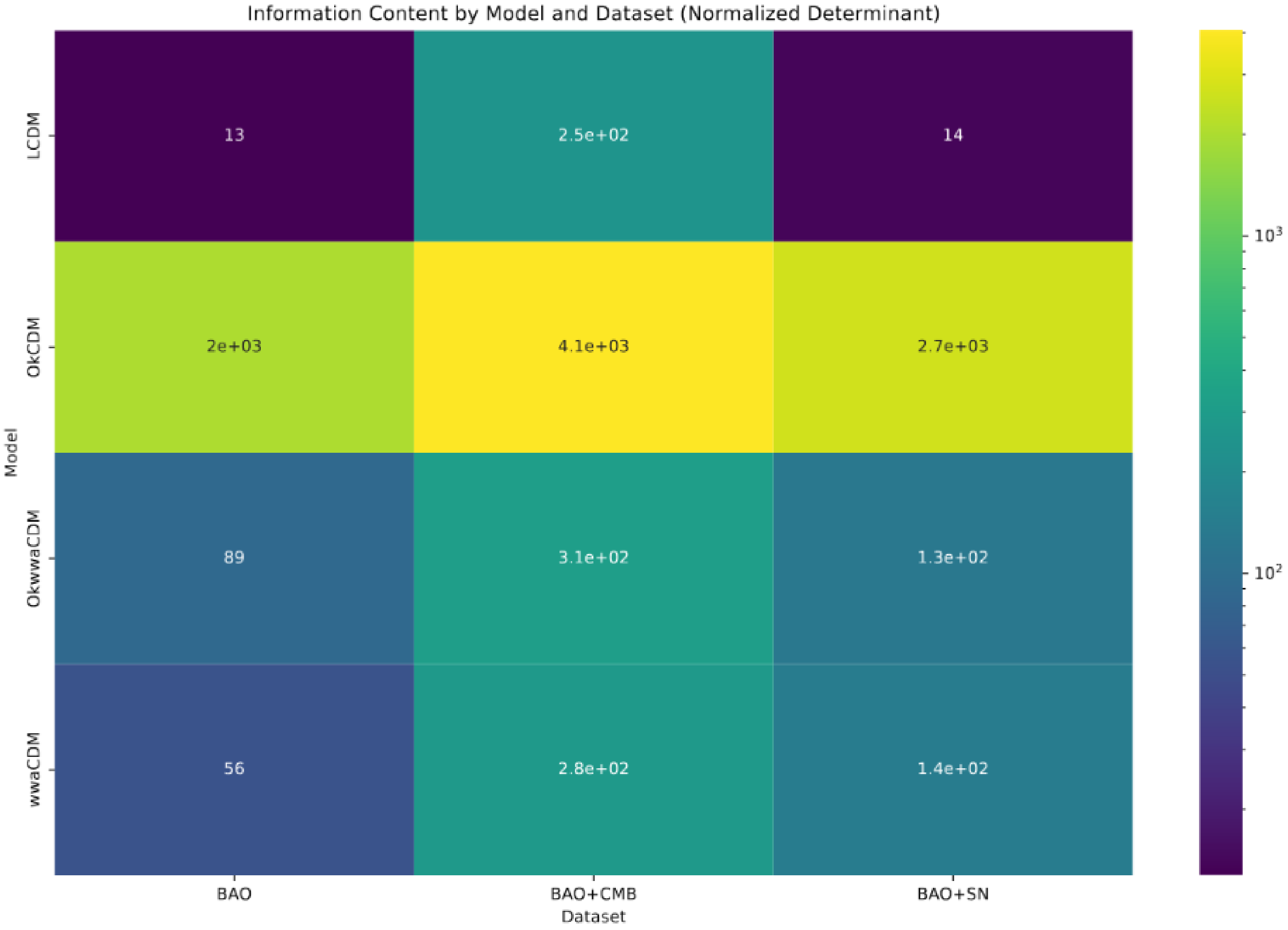
All likelihoods outside of w_0=-1







LCDM with BAO data: Method Comparison
marginalized: μ=0.295, σ=0.010
profile: μ=0.295, σ=0.009
taylor: μ=0.295, σ=0.010
full: μ=0.297, σ=0.008
OkCDM with BAO data: Method Comparison
marginalized: μ=0.291, σ=0.012
profile: μ=0.291, σ=0.012
taylor: μ=0.291, σ=0.012
full: μ=0.293, σ=0.011
OkCDM with BAO data: MCMC vs Fisher Comparison
marginalized MCMC: μ=0.291, σ=0.012
marginalized Fisher: μ=0.291, σ=0.012
profile MCMC: μ=0.291, σ=0.012
profile Fisher: μ=0.291, σ=0.012
taylor MCMC: μ=0.291, σ=0.012
taylor Fisher: μ=0.291, σ=0.012
full MCMC: μ=0.293, σ=0.011
full Fisher: μ=0.293, σ=0.011
wwaCDM with BAO data: Method Comparison
marginalized: μ=0.328, σ=0.051
profile: μ=0.323, σ=0.058
taylor: μ=0.334, σ=0.039
full: μ=0.328, σ=0.036
OkwwaCDM with BAO data: Method Comparison
marginalized: μ=0.343, σ=0.042
profile: μ=0.332, σ=0.061
taylor: μ=0.337, σ=0.053
full: μ=0.333, σ=0.036
wwaCDM with BAO data: MCMC vs Fisher Comparison
marginalized MCMC: μ=0.328, σ=0.051
marginalized Fisher: μ=0.370, σ=0.051
profile MCMC: μ=0.323, σ=0.058
profile Fisher: μ=0.371, σ=0.058
taylor MCMC: μ=0.334, σ=0.039
taylor Fisher: μ=0.370, σ=0.039
full MCMC: μ=0.328, σ=0.036
full Fisher: μ=0.364, σ=0.036
Likelihood
Ωm
DDE breaks Gaussianity



Information Content by Model and Dataset (Normalized Determinant)
LCDM
OkCDM
OkwwaCDM
wwaCDM
Model
13
2.5e+02
14
2e+03
4.1e+03
2.7e+03
89
3.1e+02
1.3e+02
56
2.8e+02
1.4e+02
BAO
BAO+CMB
BAO+SN
Dataset
10^3
10^2

Main results

- The actual posterior distribution shape for DDE models is a banana
- Muldimodality is not always well sampled in higher dimensions
- In lower dimensions most samplers are ok
- Some DDE models might require assuming GMM distributions

- Likelihood choice has minimal impact for standard models (ΛCDM, OkCDM) but becomes important for extended models
- Marginalized likelihood provides efficient approximation when full likelihood cannot be used
- DDE models exhibit non-Gaussian likelihood structure
- Dataset informativeness varies dramatically by parameter
- Some directions in DDE parameter space remain unconstrained even with combined probes
- The tension between datasets (BAO → BAO+SN → BAO+CMB) visible clearly

Methodological choices can be as significant as the physical tensions we want to quantify!

Thank you for your attention!

Credits: NASA, ESA, CSA, STScI, Webb ERO Production Team