THE EFFECTIVE RUNNING HUBBLE CONSTANT IN SNe Ia AS A MARKER FOR THE DARK ENERGY NATURE

E. Fazzari, M.G. Dainotti, G. Montani, and A. Melchiorri

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Elisa Fazzari

PhD student
Sapienza University of Rome

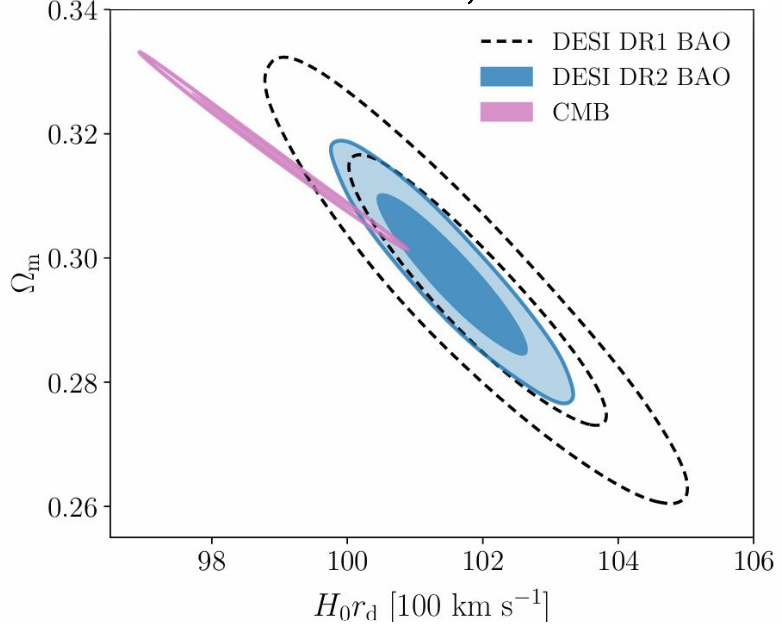
elisa.fazzari@uniroma1.it

Outlook

Evolutionary Dark Energy vs Cosmological Constant scenario:

DESI-DR2 results with a Flat-ΛCDM model:

DESI Collaboration, 2503.14738



$$\frac{D_M(z)}{r_d} = \frac{1}{H_0 r_d} \int_0^z \frac{c\,dz'}{\sqrt{\Omega_{m0}(1+z')^3+(1-\Omega_{m0})}}$$

$$\frac{D_H(z)}{r_d} = \frac{c}{r_d H_0 \sqrt{\Omega_{m0}(1+z')^3+(1-\Omega_{m0})}}$$

$$\frac{D_V(z)}{r_d} = \frac{\left(z D_M(z)^2 D_H(z)\right)^{\frac{1}{3}}}{r_d}$$

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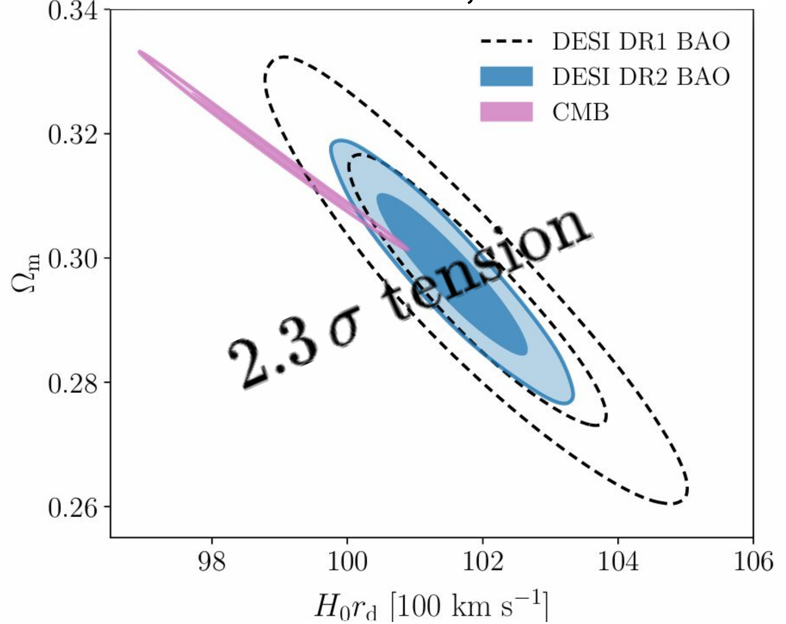

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Outlook

Evolutionary Dark Energy vs Cosmological Constant scenario:

D. Brout, D. Scolnic et al., 2202.04077

	Ω_M	Ω_Λ	H_0	w_0	w_a
Pantheon+ & SH0ES - *All Models*					
FlatΛCDM	0.334 ± 0.018	0.666 ± 0.018	73.6 ± 1.1	-	-
ΛCDM	0.306 ± 0.057	0.625 ± 0.084	73.4 ± 1.1	-	-
Flat$w$CDM	$0.309^{+0.063}_{-0.060}$	$0.691^{+0.069}_{-0.063}$	73.5 ± 1.1	-0.90 ± 0.14	-
Flatw_0w_aCDM	$0.403^{+0.054}_{-0.098}$	$0.597^{+0.098}_{-0.054}$	73.3 ± 1.1	-0.93 ± 0.15	$-0.1^{+0.9}_{-2.0}$

	Ω_M	Ω_K	w_0	w_a
DES-SN5YR (No External Priors)				
Flat ΛCDM	0.352 ± 0.017	…	…	…
ΛCDM	$0.291^{+0.063}_{-0.065}$	0.16 ± 0.16	…	…
Flat wCDM	$0.264^{+0.074}_{-0.096}$	…	$-0.80^{+0.14}_{-0.16}$	…
Flat w_0w_aCDM	$0.495^{+0.033}_{-0.043}$	…	$-0.36^{+0.36}_{-0.30}$	$-8.8^{+3.7}_{-4.5}$

DES-Y5 Collaboration, 2401.02929

Outlook

Evolutionary Dark Energy vs Cosmological Constant scenario:

$w_0 w_a$CDM vs. ΛCDM

Datasets	$\Delta\chi^2_{\rm MAP}$	Significance	Δ(DIC)
DESI	-4.7	1.7σ	-0.8
DESI+$(\theta_*, \omega_{\rm b}, \omega_{\rm bc})_{\rm CMB}$	-8.0	2.4σ	-4.4
DESI+CMB (no lensing)	-9.7	2.7σ	-5.9
DESI+CMB	-12.5	3.1σ	-8.7
DESI+Pantheon+	-4.9	1.7σ	-0.7
DESI+Union3	-10.1	2.7σ	-6.0
DESI+DESY5	-13.6	3.3σ	-9.3
DESI+DESY3 (3×2pt)	-7.3	2.2σ	-2.8
DESI+DESY3 (3×2pt)+DESY5	-13.8	3.3σ	-9.1
DESI+CMB+Pantheon+	-10.7	2.8σ	-6.8
DESI+CMB+Union3	-17.4	3.8σ	-13.5
DESI+CMB+DESY5	-21.0	4.2σ	-17.2

DESI Collaboration, 2503.14738

Are SNe Ia alone really unable to reveal the presence of Evolutionary Dark Energy and not discriminate its nature?

Theory part

What is the effective running Hubble constant?

Similar diagnostic tool for DE:
DESI Collaboration, 2503.14743
Sahni et al., 0807.3548

A new diagnostic for Dark Energy:

$$H(z) = \mathcal{H}_0(z) E(z)^{\Lambda\mathrm{CDM}} \longrightarrow \mathcal{H}_0(z) = H_0 \frac{E(z)}{E(z)^{\Lambda\mathrm{CDM}}}$$

Dainotti et al., 2103.02117
Kazantzidis et al., 2004.02155
Jia et al., 2212.00238

arXiv:2506.04162

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$$\mathcal{H}_0(z) \equiv H_0 \qquad \Lambda\mathrm{CDM}$$

$$\mathcal{H}_0(z) \equiv H_0 \sqrt{\frac{\Omega_m^0 (1+z)^3 + (1-\Omega_m^0)(1+z)^{3(1+w)}}{\Omega_m^0 (1+z)^3 + (1-\Omega_m^0)}} \qquad w\mathrm{CDM}$$

$$\mathcal{H}_0(z) \equiv H_0 \sqrt{\frac{\Omega_m^0 (1+z)^3 + (1-\Omega_m^0)(1+z)^{3(1+w_0+w_a)} e^{-3w_a \frac{z}{1+z}}}{\Omega_m^0 (1+z)^3 + (1-\Omega_m^0)}} \qquad w_0 w_a\mathrm{CDM}$$

Dainotti et al., 2103.02117
Kazantzidis et al., 2004.02155
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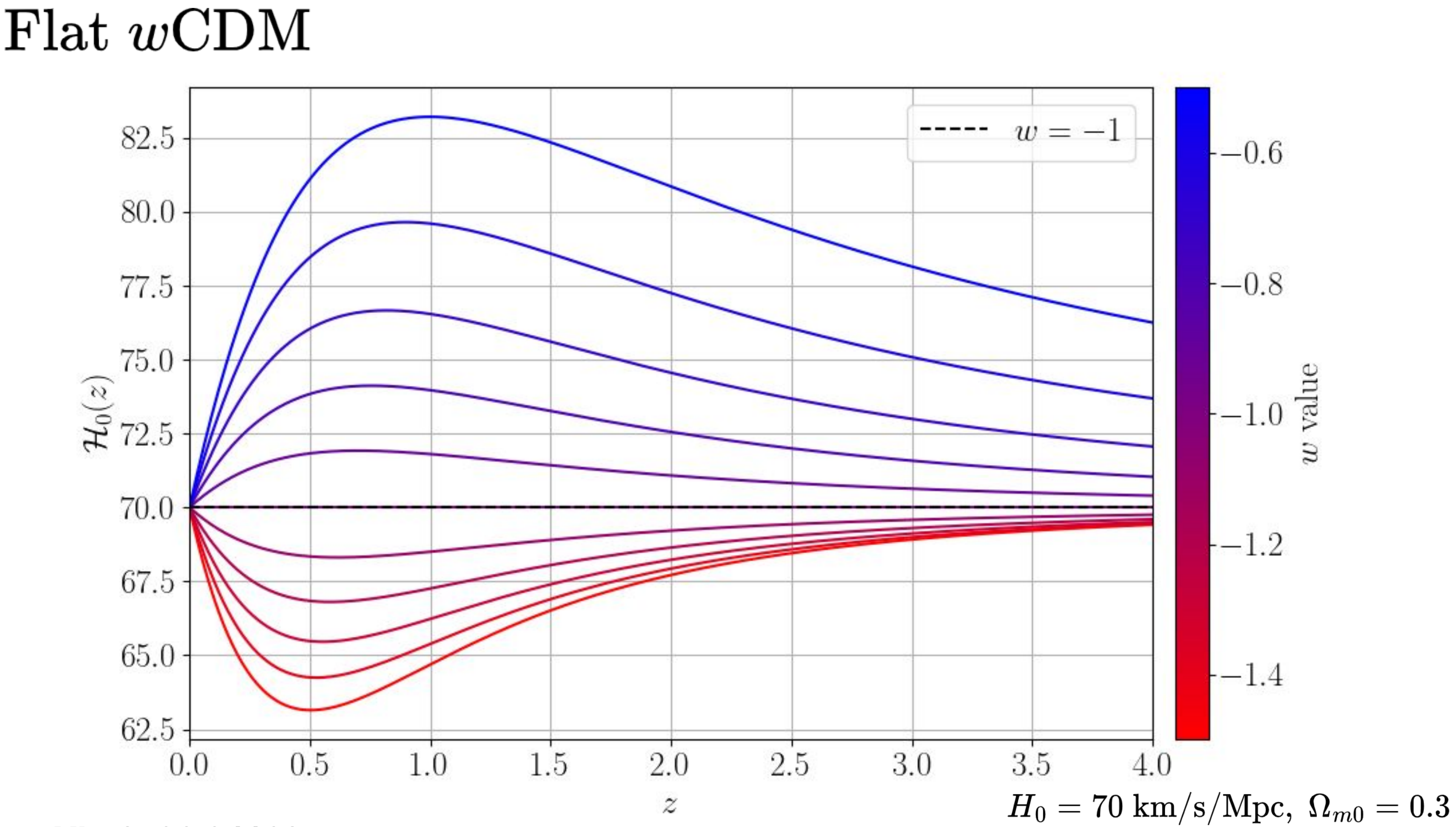
arXiv:2506.04162

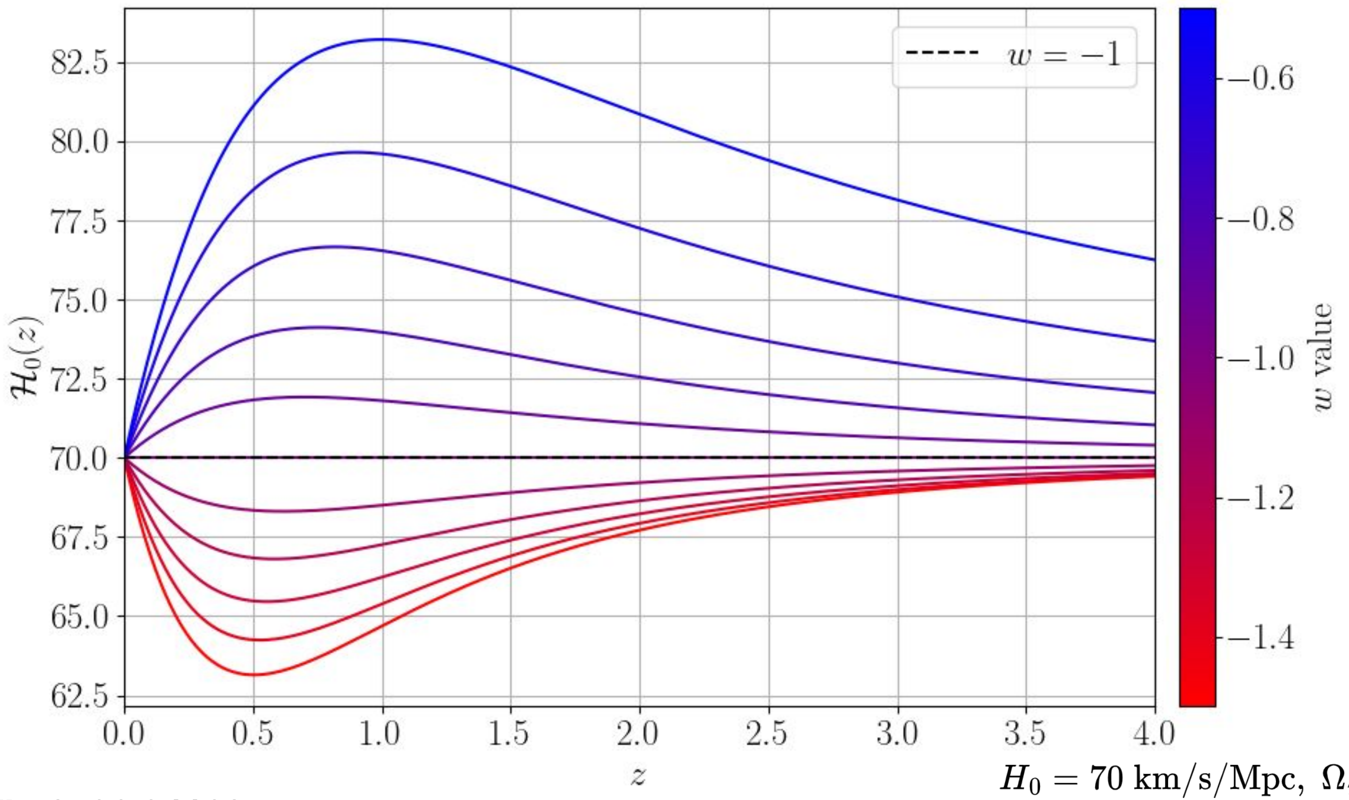
Effective running Hubble constant



arXiv:2506.04162

Effective running Hubble constant

Flat wCDM



Quintessence

Phantom

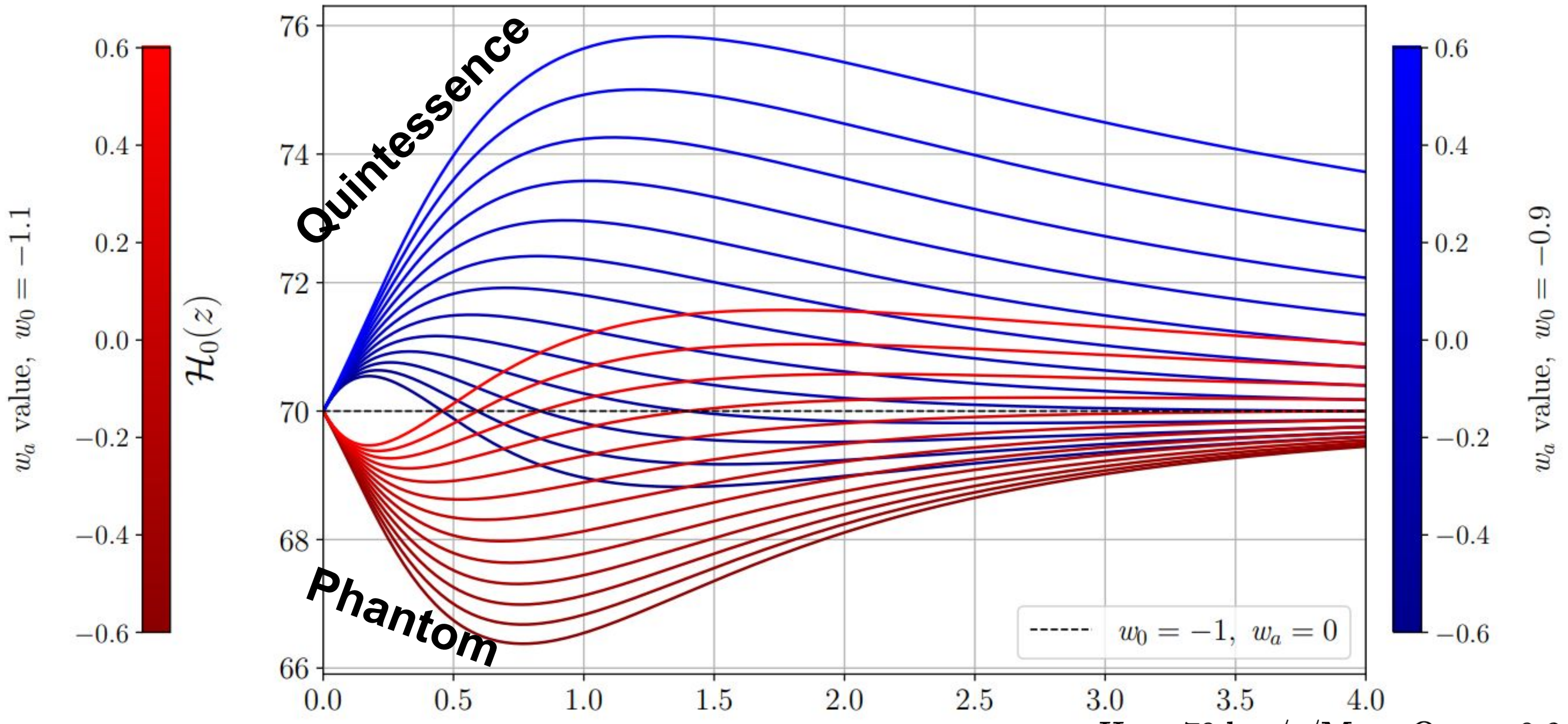
$H_0 = 70$ km/s/Mpc, $\Omega_{m0} = 0.3$

arXiv:2506.04162

Effective running Hubble constant

Flat $w_0 w_a$CDM

$$\mathcal{H}_0'^{w_0 w_a}(z=0) = \frac{3}{2} H_0 (1 - \Omega_{m0})(1 + w_0)$$



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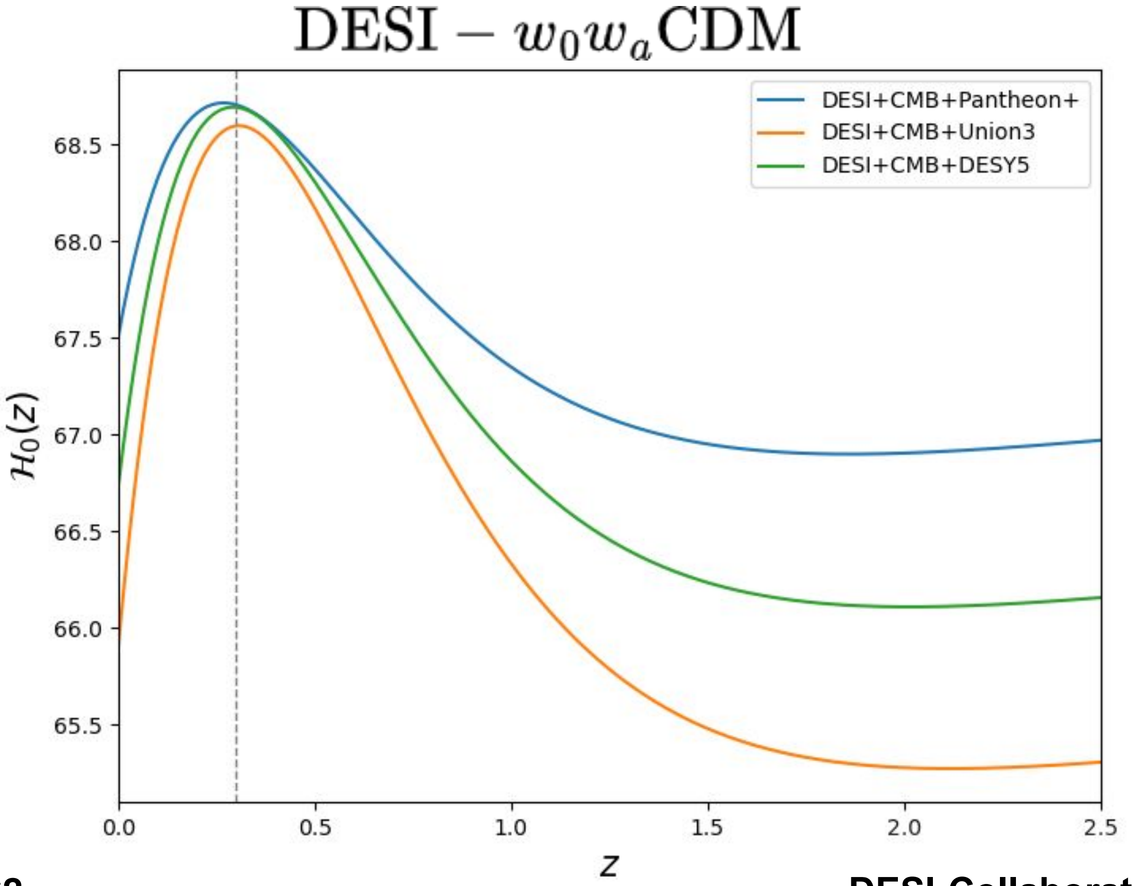
Effective running Hubble constant



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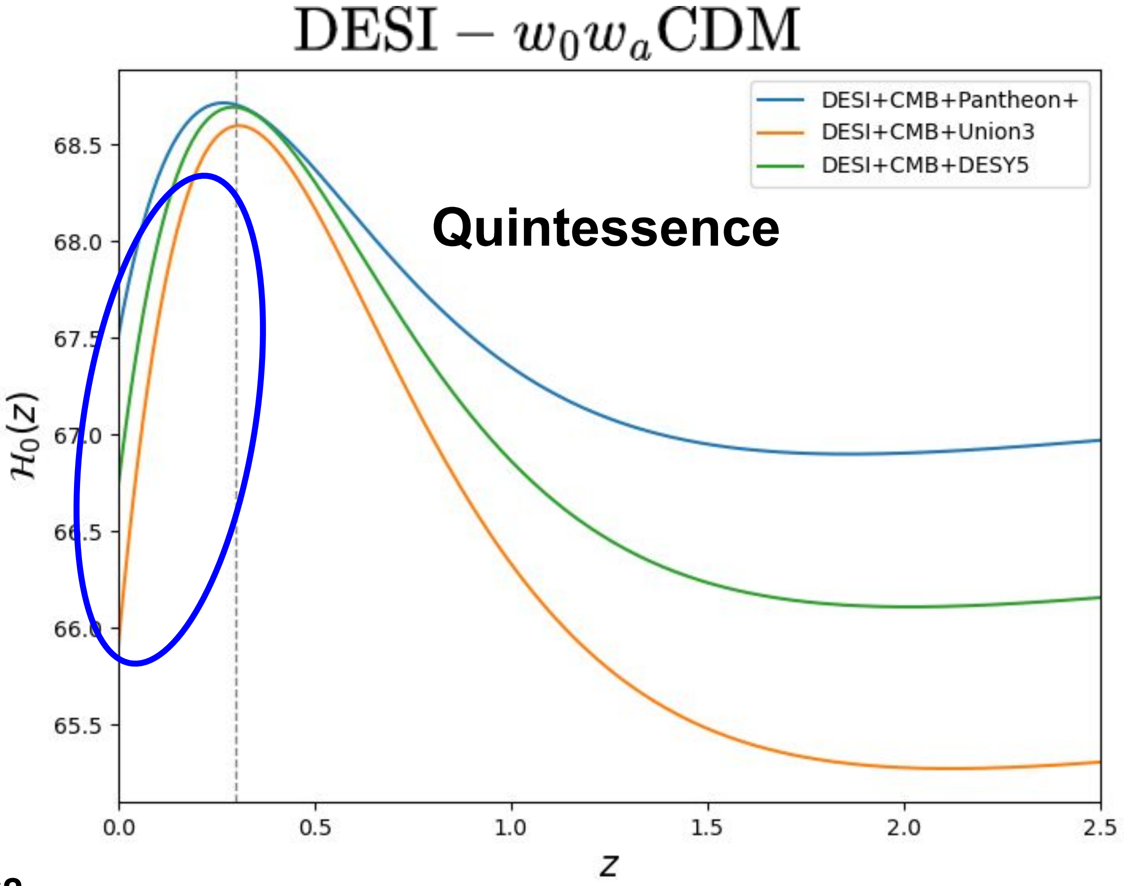
DESI-Collaboration, 2503.14738

Effective running Hubble constant



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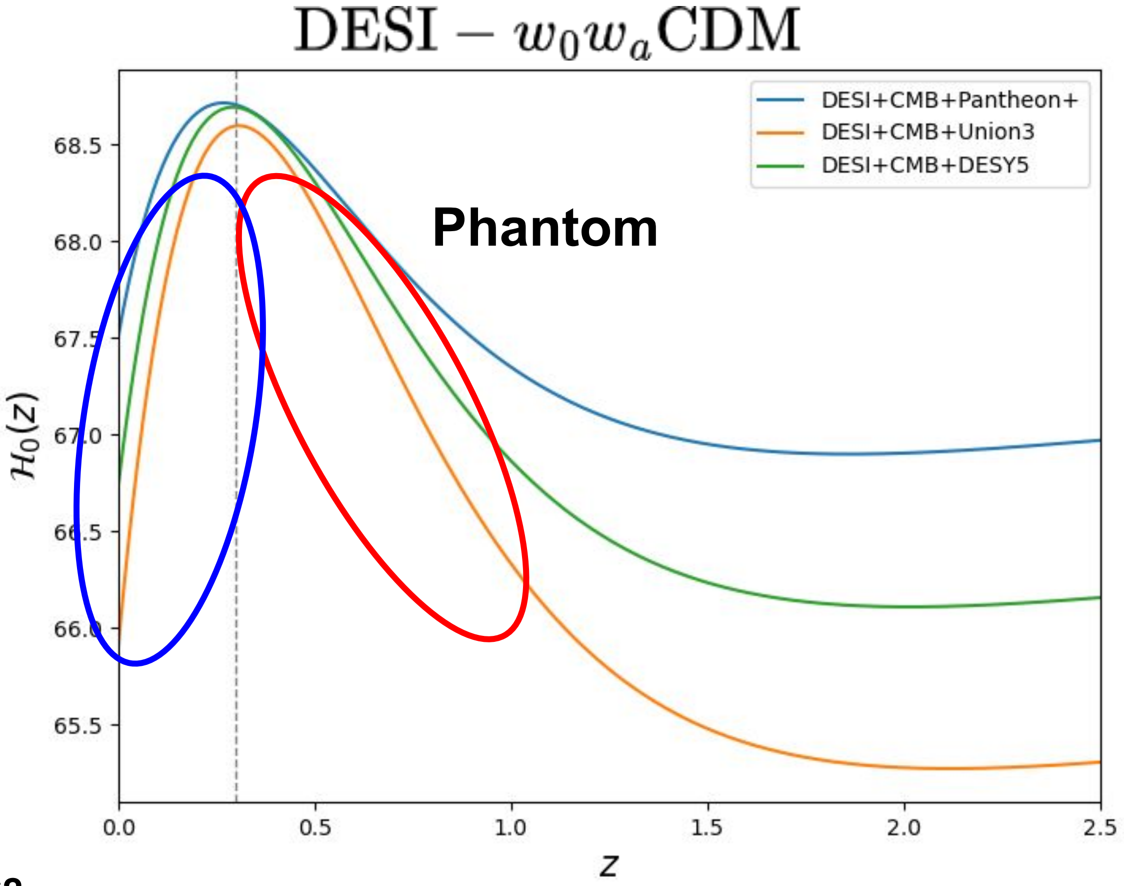
Effective running Hubble constant



arXiv:2506.04162

Dark energy models analyzed

1. $w\mathrm{CDM}$

$$\mathcal{H}_0(z) \equiv H_0 \sqrt{\frac{\Omega_m^0(1+z)^3+(1-\Omega_m^0)(1+z)^{3(1+w)}}{\Omega_m^0(1+z)^3+(1-\Omega_m^0)}}$$

2. $\mathrm{CPL}'(w_0 = w_a)$

$$\mathcal{H}_0(z) \equiv H_0 \sqrt{\frac{\Omega_m^0(1+z)^3+(1-\Omega_m^0)(1+z)^{3(1+2w)}e^{-3w\frac{z}{1+z}}}{\Omega_m^0(1+z)^3+(1-\Omega_m^0)}}$$

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3. Dynamical Dark Energy (DDE) model

   Dark energy created by the variation of cosmological gravitational field

arXiv:2506.04162

Dark energy models analyzed

Dynamical Dark Energy (DDE) model

- **Particle creation mechanism**

$$\rho\prime_{de}(z) = \frac{3H}{(1+z)}[(1+w_{de})\rho_{de} + p_c]$$

$$3Hp_c \equiv -\Gamma\,(1+w_{de})\rho_{de}$$

phenomenological constant
particle creation rate

Lima et al., 0708.3397
Montani., 0101113
Elizalde et al., 2407.20285

arXiv:2506.04162

Dark energy models analyzed

Dynamical Dark Energy (DDE) model

- **Particle creation mechanism**

$$\rho'_{de}(z) = \frac{3H}{(1+z)}\left[(1+w_{de})\rho_{de} + p_c\right]$$

$$3Hp_c \equiv -\Gamma\,(1+w_{de})\rho_{de}$$

phenomenological constant
particle creation rate

$$\gamma_0 \equiv \Gamma/H_0$$

$$\rho'_{de}(z) = \frac{3}{1+z}(1+w_{de})\left(1-\frac{\gamma_0}{3E(z)}\right)\rho_{de}$$

Lima et al., 0708.3397
Montani., 0101113
Elizalde et al., 2407.20285

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Dark energy models analyzed

Dynamical Dark Energy (DDE) model

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$$3Hp_c \equiv -\Gamma\,(1+w_{de})\rho_{de}$$

$$\rho'_{de}(z) = \frac{3}{1+z}(1+w_{de})\left(1 - \frac{\gamma_0}{3E(z)}\right)\rho_{de}$$

$$\gamma_0 \equiv \Gamma/H_0$$

- **Theoretical constrain**

$$q_0 \equiv -1 + \frac{1}{2}(E^2)'\big|_{z=0} = q_0^{\Lambda\mathrm{CDM}} \longrightarrow \gamma_0 = 3$$

Lemos et al., 1806.06781
Efstathiou, 2103.08723

arXiv:2506.04162

Dark energy models analyzed

Dynamical Dark Energy (DDE) model

$$E^2(z) = \Omega_m^0(1+z)^3 + \left(1-\Omega_m^0\right)(1+z)^{3(1+w)} \exp\left\{-3(1+w)\int_0^z \frac{dy}{(1+y)E(y)}\right\}$$

$$w_{eff}(z) = w - \frac{1+w}{E(z)}$$

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Dark energy models analyzed

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$$w_{eff}(z) = w - \frac{1+w}{E(z)}$$

$$\downarrow$$

$$\mathcal{H}_0(z) \equiv H_0\sqrt{\frac{\Omega_m^0(1+z)^3+(1-\Omega_m^0)(1+z)^{3(1+w)}\exp\left\{-3(1+w)\int_0^z \frac{dy}{(1+y)E(y)}\right\}}{\Omega_m^0(1+z)^3+(1-\Omega_m^0)}}$$

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Recap of models

1. wCDM

$$\mathcal{H}_0(z) \equiv H_0 \sqrt{\frac{\Omega_m^0(1+z)^3+(1-\Omega_m^0)(1+z)^{3(1+w)}}{\Omega_m^0(1+z)^3+(1-\Omega_m^0)}}$$

2. $\mathrm{CPL}'(w_0 = w_a)$

$$\mathcal{H}_0(z) \equiv H_0 \sqrt{\frac{\Omega_m^0(1+z)^3+(1-\Omega_m^0)(1+z)^{3(1+2w)}e^{-3w\frac{z}{1+z}}}{\Omega_m^0(1+z)^3+(1-\Omega_m^0)}}$$

3. DDE

$$\mathcal{H}_0(z) \equiv H_0 \sqrt{\frac{\Omega_m^0(1+z)^3+(1-\Omega_m^0)(1+z)^{3(1+w)}\exp\left\{-3(1+w)\int_0^z \frac{dy}{(1+y)E(y)}\right\}}{\Omega_m^0(1+z)^3+(1-\Omega_m^0)}}$$

ΛCDM

$$\mathcal{H}_0(z) \equiv H_0$$

Power law (PL)

$$\mathcal{H}_0(z) \equiv \frac{H_0}{(1+z)^{\alpha}}$$

Dainotti et al., 2103.02117

arXiv:2506.04162

Model testing part

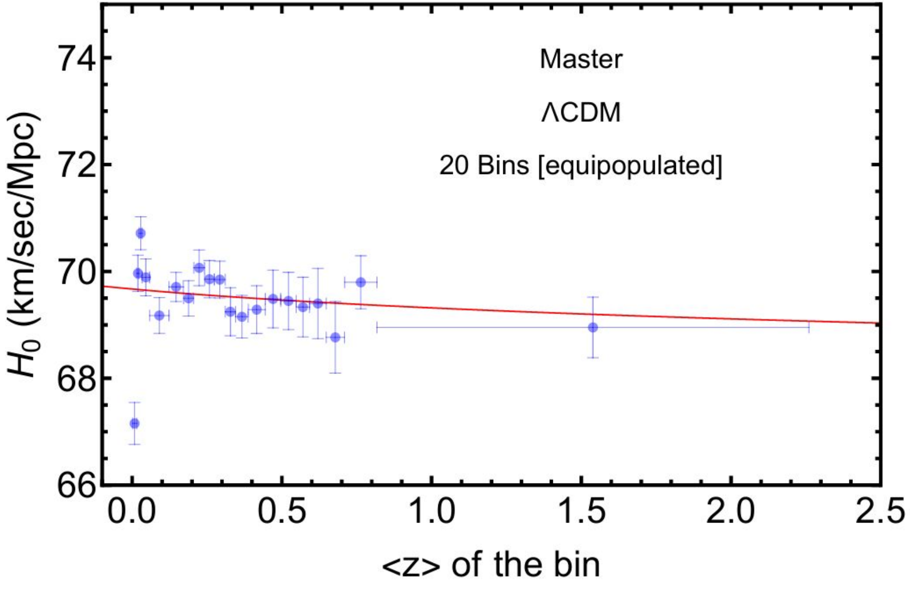
Method - Datasets

Dainotti et al., 2201.09848
Dainotti et al., 2501.11772

Binned SNe Ia datasets:

- **Pantheon binned** SNe Ia sample

- **Master binned** SNe Ia sample:
  SNe Ia from DESY5, Pantheon+,
  Pantheon and JLA
  without duplicates



arXiv:2506.04162

Method - Datasets

- Same priors as those used to obtain the binned data samples

Prior for different datasets			
Parameter	Background	Pantheon	Master
Ω_{m0}	$\mathcal{U}[0.01, 0.99]$	$\mathcal{N}[0.298, 0.022]$	$\mathcal{N}[0.322, 0.025]$
H_0 (km/s/Mpc)	$\mathcal{U}[20, 100]$	$\mathcal{N}[70.393 \pm 2.158]$	$\mathcal{U}[60, 80]$
w	$\mathcal{U}[-7, 3]$	$\mathcal{U}[-7, 3]$	$\mathcal{U}[-7, 3]$

arXiv:2506.04162

Method - Datasets

- MCMC performed with Cobaya

$$\chi^2_{tot} = \chi^2_{H_{0_{bin}}} + \chi^2_{\Omega_{m0_{bin}}}$$

- **Backy** [Giarè, Fazzari, in prep.]

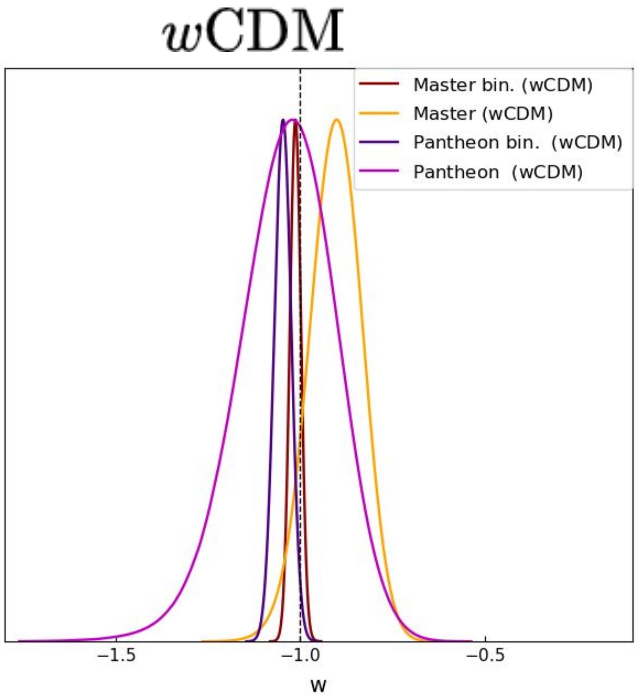
  code interfaced with Cobaya for testing late-time modified cosmological models using background observables

  mostly useful for Dark Energy and Hubble tension

arXiv:2506.04162

Results

- w constrained with higher precision from binned SNe Ia compared to the full SNe Ia dataset



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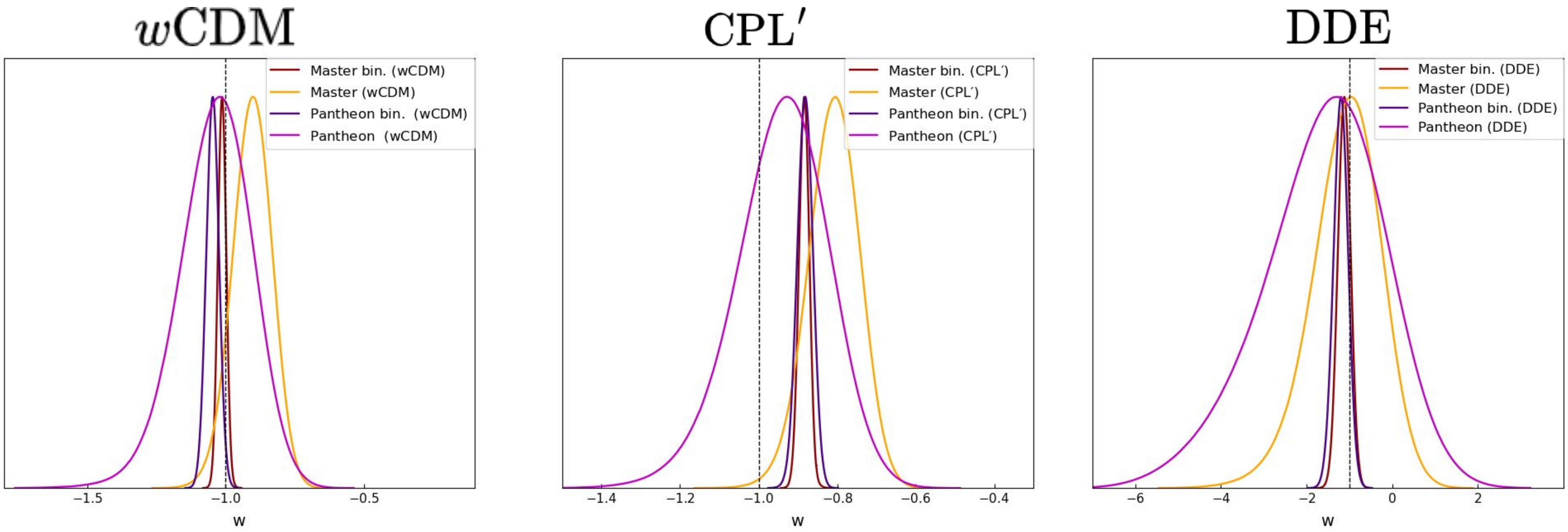

arXiv:2506.04162

Results

		Datasets			
		Master bin.	**Pantheon bin.**	**Master**	**Pantheon**
wCDM	H_0	69.865 ± 0.083	70.43 ± 0.24	69.11 ± 0.40	70.07 ± 0.43
	Ω_{m0}	0.3245 ± 0.0053	0.2978 ± 0.0047	0.318 ± 0.024	0.306 ± 0.039
	w	-1.012 ± 0.014	-1.047 ± 0.022	-0.904 ± 0.067	$-1.03^{+0.14}_{-0.12}$
CPL′	H_0	69.609 ± 0.078	69.77 ± 0.22	68.92 ± 0.39	69.81 ± 0.43
	Ω_{m0}	0.3257 ± 0.0053	0.2992 ± 0.0049	0.328 ± 0.023	0.322 ± 0.036
	w	-0.884 ± 0.012	-0.882 ± 0.019	-0.808 ± 0.061	$-0.94^{+0.11}_{-0.10}$
DDE	H_0	69.838 ± 0.061	70.10 ± 0.14	69.50 ± 0.30	69.99 ± 0.31
	Ω_{m0}	0.3247 ± 0.0054	0.2979 ± 0.0050	0.343 ± 0.024	0.311 ± 0.041
	w	-1.12 ± 0.14	-1.20 ± 0.16	$-1.06^{+0.85}_{-0.72}$	$-1.6^{+1.6}_{-1.3}$

arXiv:2506.04162

Results

- $w\mathrm{CDM}$ and DDE perform better compared to the CPL'

$$\ln B_{i,ref} = \ln B_{ref} - \ln B_i$$

$$2.5 < \ln B_{i,ref} > 5.0$$

moderate

	Master bin.		Pantheon bin.	
Model	$\ln \mathrm{B}_{i,\Lambda\mathrm{CDM}}$	$\ln \mathrm{B}_{i,PL}$	$\ln \mathrm{B}_{i,\Lambda\mathrm{CDM}}$	$\ln \mathrm{B}_{i,PL}$
*w*CDM	4.64	6.02	2.43	4.65
CPL'	14.51	15.90	14.52	16.73
DDE	3.09	4.48	2.56	4.77
PL	-1.38	—	-2.21	—

Jeffreys et al., 1998
Trotta, 0803.4089
Giarè, wgcosmo GitHub

arXiv:2506.04162

Results

- wCDM and DDE perform better compared to the CPL'
- CPL' model is significantly disfavored with respect to ΛCDM

$$\ln B_{i,ref} = \ln B_{ref} - \ln B_i$$

$$\ln B_{i,ref} > 5 \text{ strong}$$

	Master bin.		Pantheon bin.	
Model	$\ln \mathrm{B}_{i,\Lambda\mathrm{CDM}}$	$\ln \mathrm{B}_{i,PL}$	$\ln \mathrm{B}_{i,\Lambda\mathrm{CDM}}$	$\ln \mathrm{B}_{i,PL}$
wCDM	4.64	6.02	2.43	4.65
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Jeffreys et al., 1998
Trotta, 0803.4089
Giarè, wgcosmo GitHub

arXiv:2506.04162

Results

- wCDM and DDE perform better compared to the CPL'
- CPL' model is significantly disfavored with respect to ΛCDM
- PL model emerges as the favored one

$$\ln B_{i,ref} = \ln B_{ref} - \ln B_i$$

$$1.0 < \ln B_{i,ref} > 2.5 \ \ \text{weak}$$

	Master bin.		Pantheon bin.	
Model	$\ln \mathrm{B}_{i,\Lambda\mathrm{CDM}}$	$\ln \mathrm{B}_{i,PL}$	$\ln \mathrm{B}_{i,\Lambda\mathrm{CDM}}$	$\ln \mathrm{B}_{i,PL}$
wCDM	4.64	6.02	2.43	4.65
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arXiv:2506.04162

Method - Datasets

Background analysis:

- Pantheon and Master SNe Ia
- DESI DR2 calibrated with Planck constraint on $r_d = \mathcal{N}[147.09, 0.26]$
- Cosmic Chronometer

Prior for different datasets			
Parameter	Background	Pantheon	Master
Ω_{m0}	$\mathcal{U}[0.01, 0.99]$	$\mathcal{N}[0.298, 0.022]$	$\mathcal{N}[0.322, 0.025]$
H_0 (km/s/Mpc)	$\mathcal{U}[20, 100]$	$\mathcal{N}[70.393 \pm 2.158]$	$\mathcal{U}[60, 80]$
w	$\mathcal{U}[-7, 3]$	$\mathcal{U}[-7, 3]$	$\mathcal{U}[-7, 3]$

arXiv:2506.04162

Results

- Statistical uncertainties larger than those obtained from the binned SNe Ia data alone

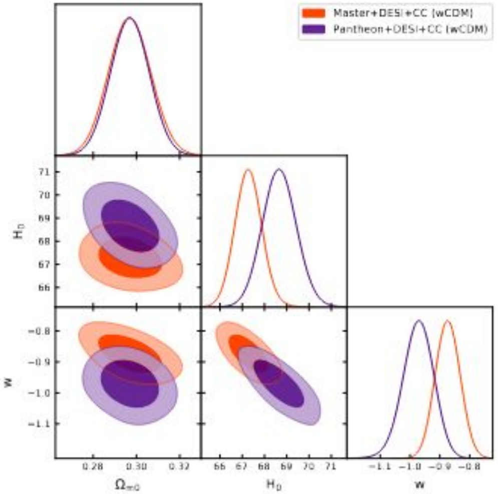
- No 1σ discrimination between phantom and quintessence regimes for the wCDM and DDE models

		Datasets	
		Master+DESI+CC	**Pantheon+DESI+CC**
wCDM	H_0	67.27 ± 0.57	68.67 ± 0.72
	Ω_{m0}	0.2969 ± 0.0092	0.2970 ± 0.0084
	w	-0.873 ± 0.039	-0.972 ± 0.048
CPL′	H_0	66.95 ± 0.55	68.14 ± 0.66
	Ω_{m0}	0.3230 ± 0.0080	0.3185 ± 0.0077
	w	-0.777 ± 0.032	-0.845 ± 0.039
DDE	H_0	68.36 ± 0.45	69.12 ± 0.51
	Ω_{m0}	$0.305^{+0.015}_{-0.013}$	$0.284^{+0.028}_{-0.020}$
	w	-0.90 ± 0.28	-0.79 ± 0.43
ΛCDM	H_0	68.37 ± 0.46	69.01 ± 0.47
	Ω_{m0}	0.3102 ± 0.0079	0.2977 ± 0.0077

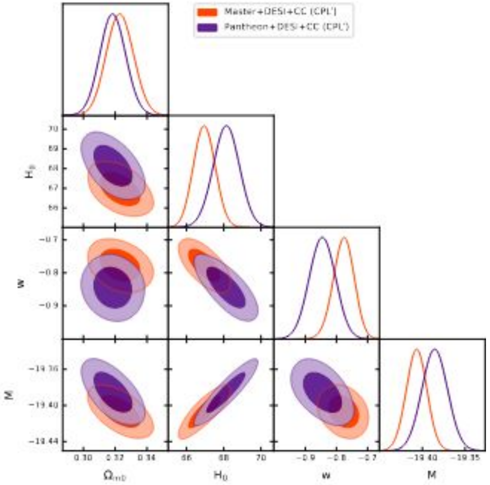
arXiv:2506.04162

Results

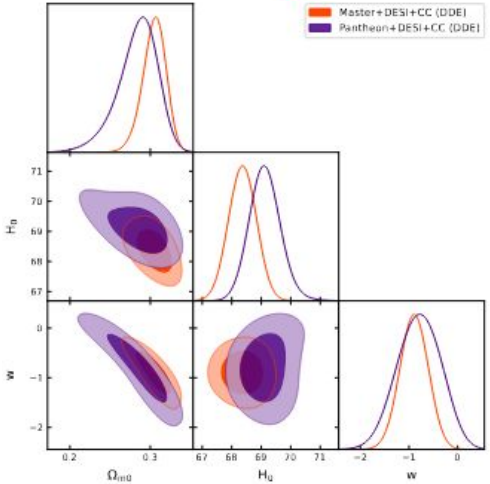
$w\mathrm{CDM}$



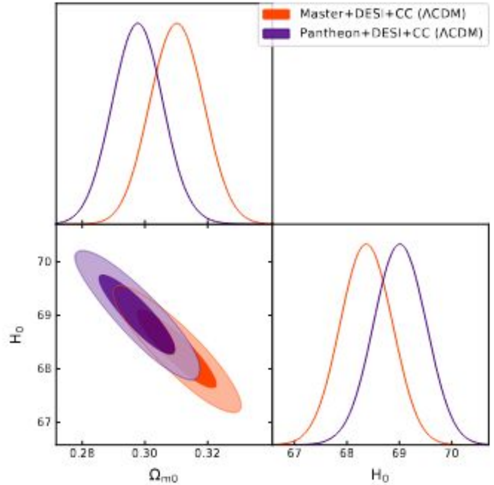
CPL'



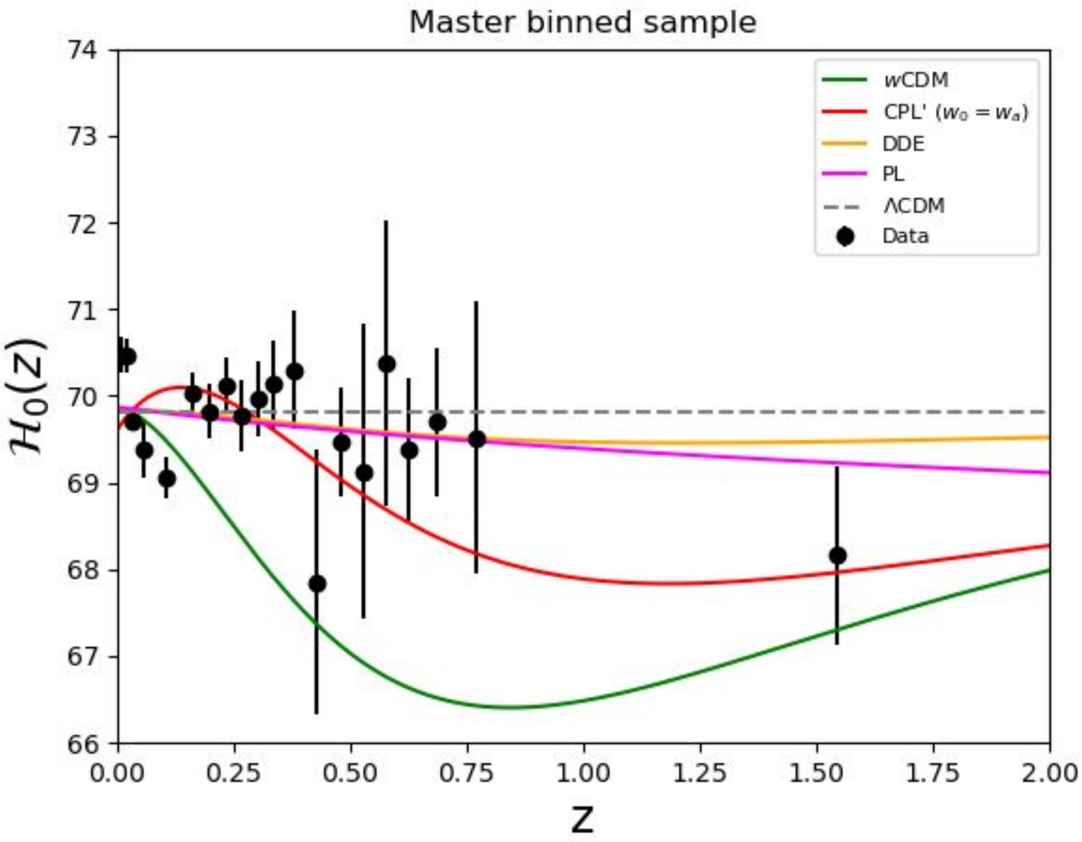
DDE



$\Lambda\mathrm{CDM}$



Effective running Hubble constant



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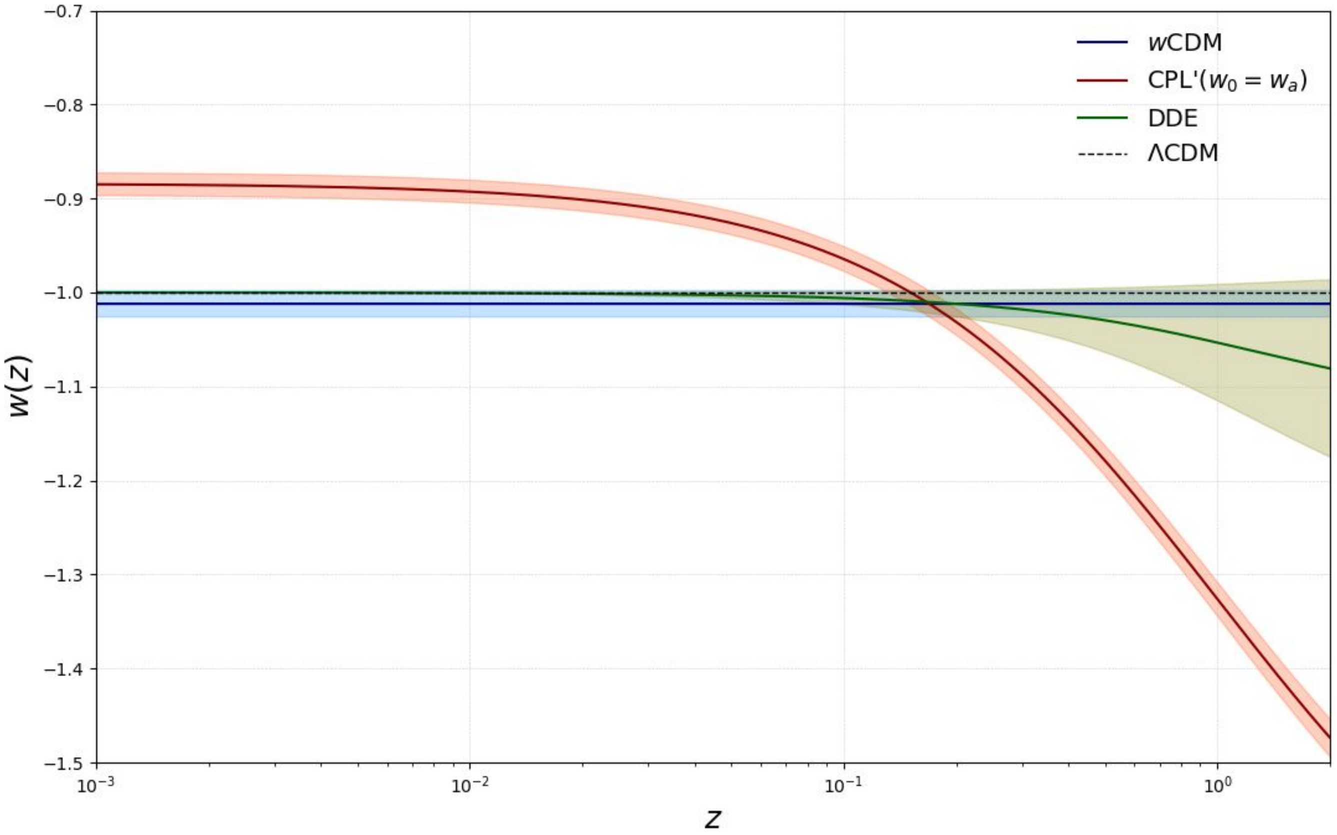
Dark Energy EoS



arXiv:2506.04162

Take home messages

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Thank you

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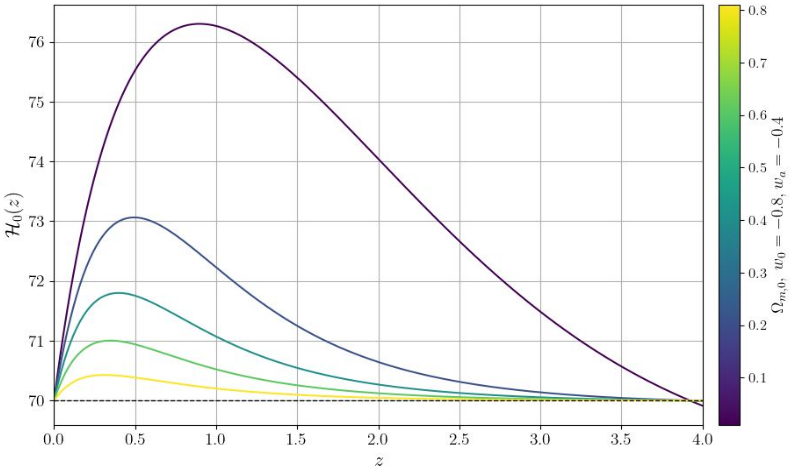
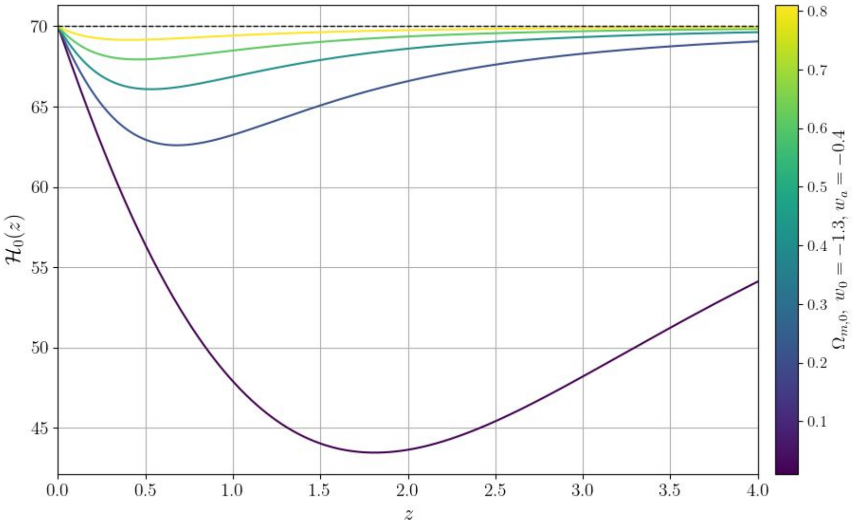
Backup slides



Phantom

Quintessence



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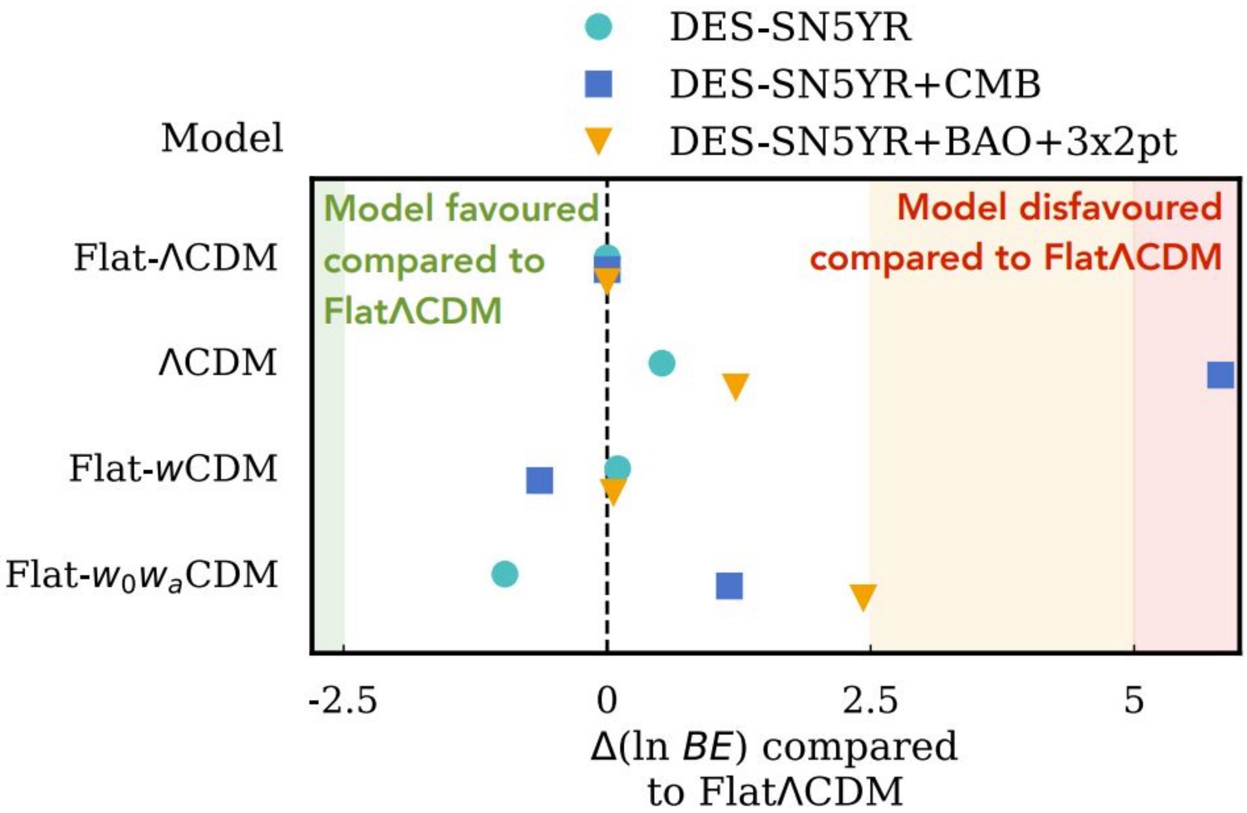
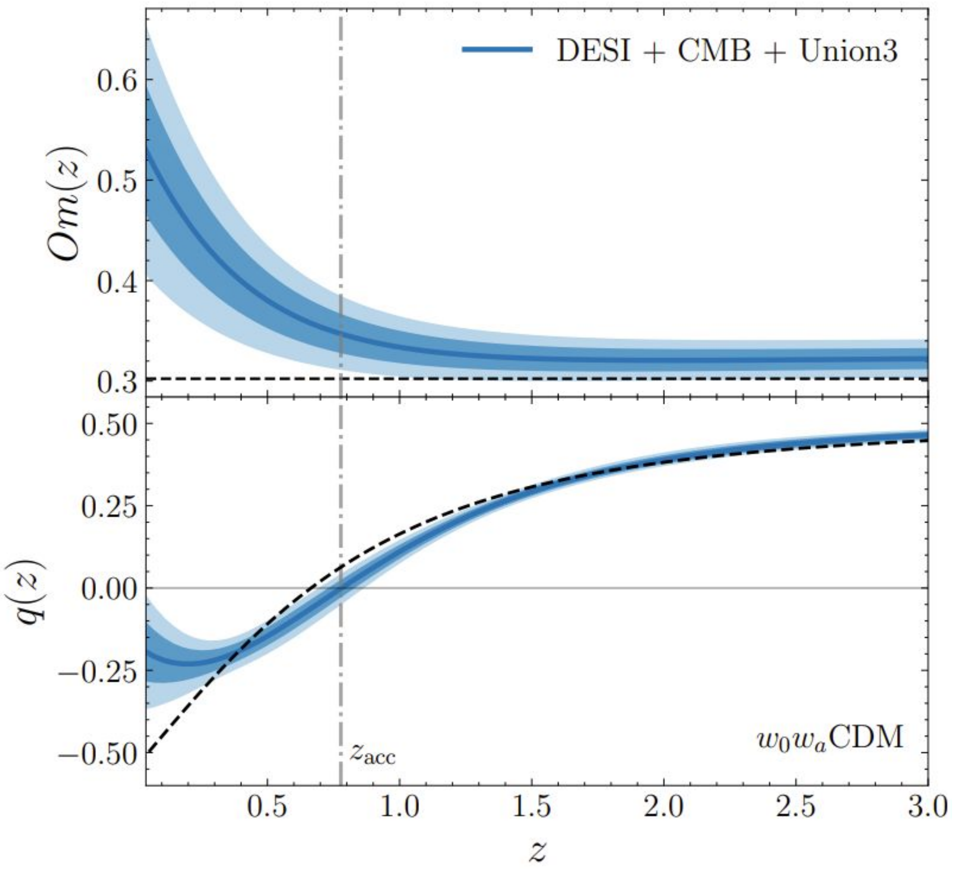


Backup slides

$$Om(z) = \frac{h^2(z)-1}{(1+z)^3-1}$$

$$h^2(z) = \frac{H^2(z)}{H_0^2}$$



DESI Collaboration, 2503.14743
Sahni et al., 0807.3548