The Ph-LsCDM Model: Towards a Solution to the H0 tension Problem

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The CosmoVerse White Paper

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Cosmological tensions

Significant descrepancies in cosmological data obtained in different experiments!

- **H_0- present day Hubble parameter**
- **S_8-growth matter density parameter**
- **M_B-type Ia Supernova absolute magnitude**
- **BAO Lyman-α**

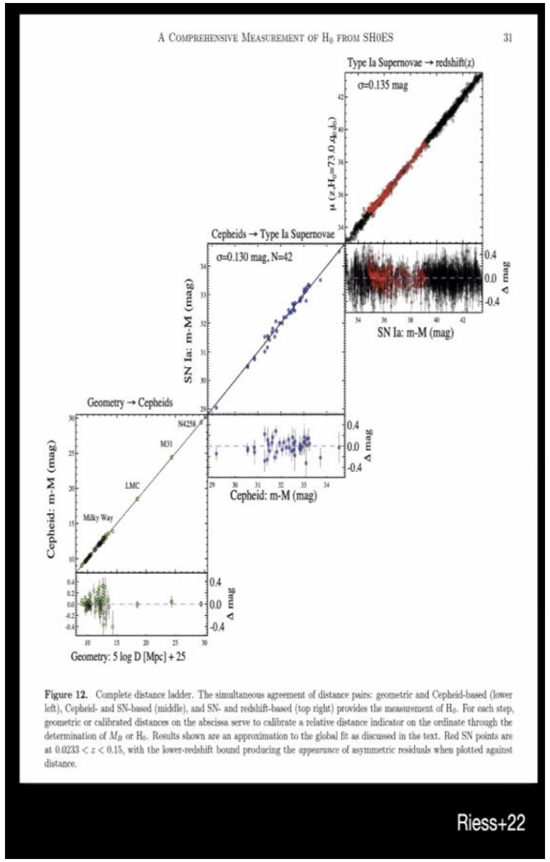
E.g., H_0- tension:

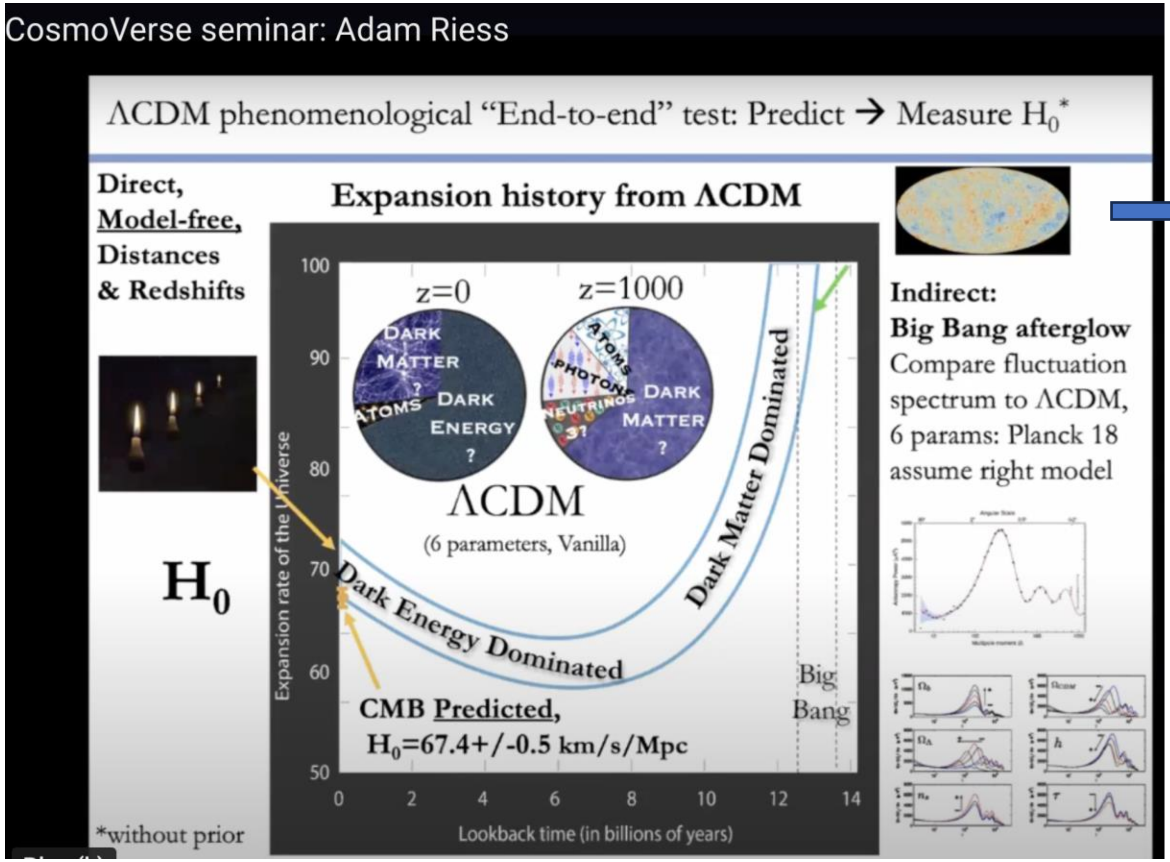
5 σ deviation!

H_0= 73.04 km s^{-1} Mpc $^{-1}$ – Cepheid-calibrated distance ladder approach (SHOES team).

H_0= 67.4 km s^{-1} Mpc $^{-1}$ – CMB measurements assuming ΛCDM (Planck team).

SHOES ladder:





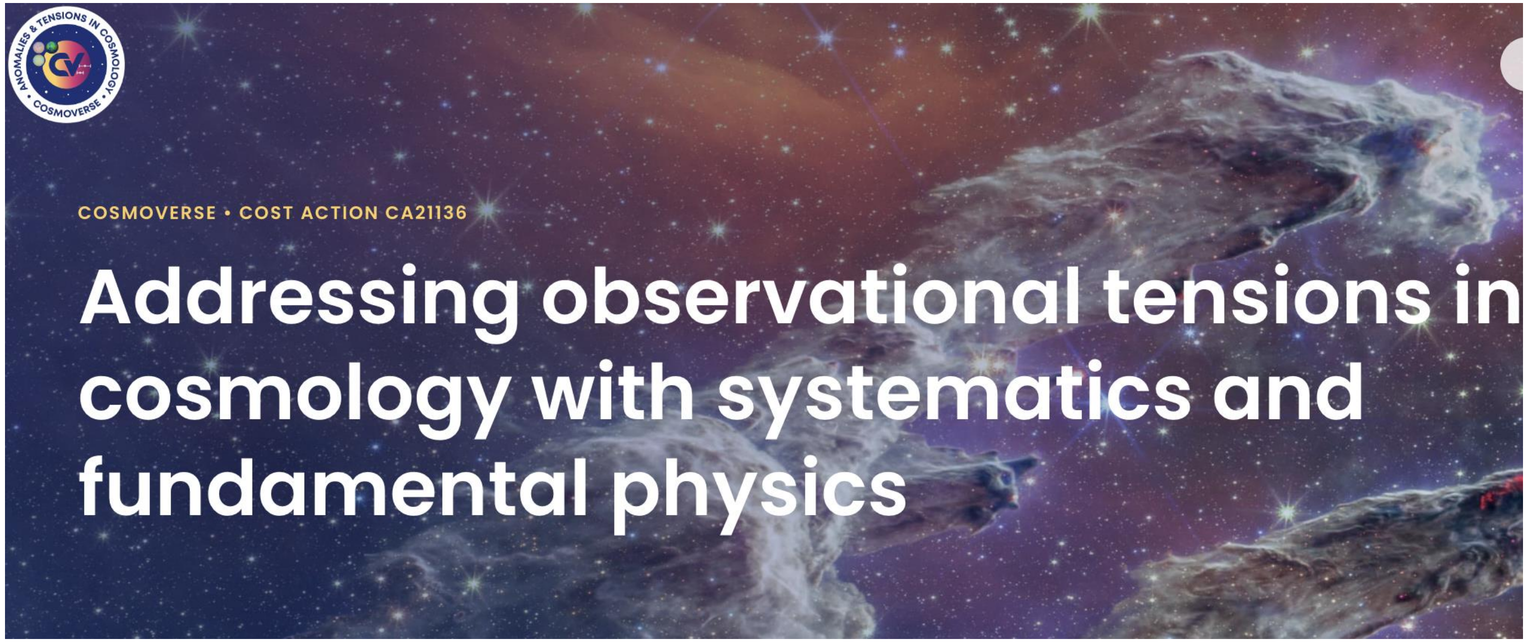
comoving angular diameter distance to last scattering:

$$D_{\rm m}(z_*) = \int_0^{z_*} c\frac{dz}{H} = 13872.83\ \text{Mpc}\,.$$

ΛCDM Planck18

H_0 is estimated from this expression

How to resolve these tensions?



This program brings together cosmologists to solve this problem.

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arXiv:2504.01669 [pdf, other]

The CosmoVerse White Paper: Addressing observational tensions in cosmology with systematics and fundamental physics

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Comments: Comments are welcome by the community, 416 pages, 81 figures

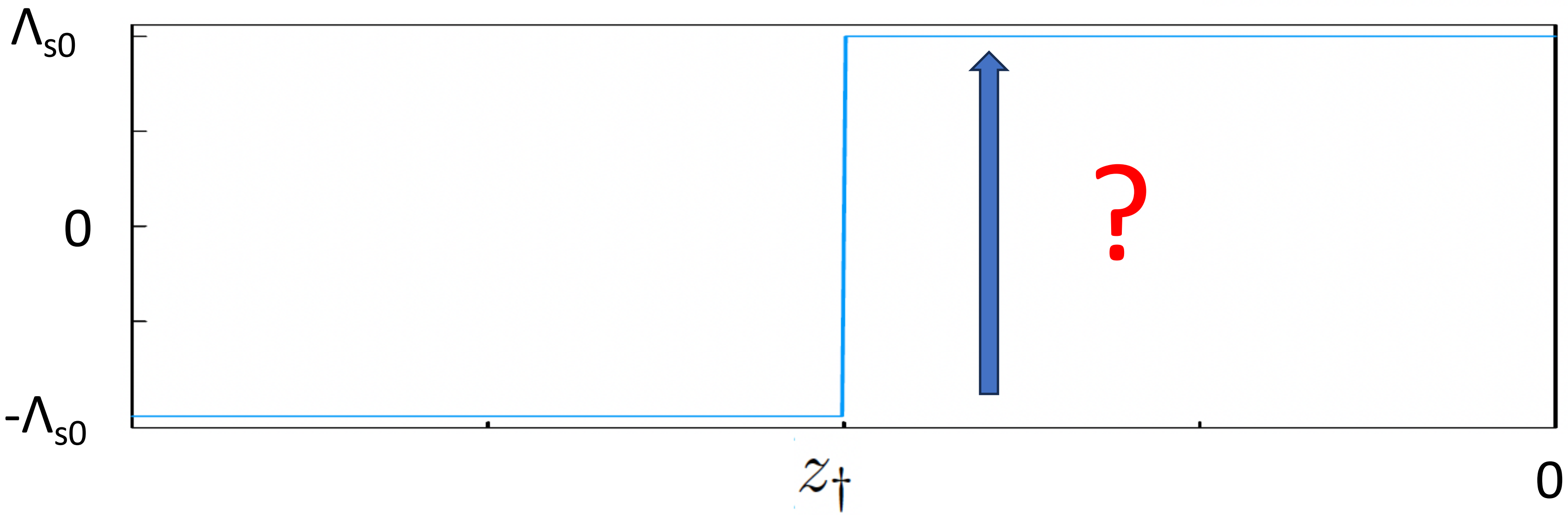
Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO)**; General Relativity and Quantum Cosmology (gr-qc); High Energy Physics - Phenomenology (hep-ph)

The ΛsCDM model emerges as one of the promising models for addressing major cosmological tensions, viz., H_0, M_B (Type Ia Supernovae absolute magnitude), and S_8 (growth parameter) tensions, along with some other less significant tensions, and stands as the most economical model with this capability.

Phys. Rev. D 101, 063528 (2020);
Phys. Rev. D 104, 123512 (2021);
Phys. Rev. D 108, 023513 (2023);
arXiv:2307.10899 **[astro-ph.CO]**

O. Akarsu, J. D. Barrow, L. A. Escamilla, S. Kumar, E. Özülker, J. A. Vazquez, A. Yadav, E. Di Valentino, R. C. Nunes

Abrupt mirror AdS-dS transition: $\Lambda \quad \rightarrow \quad \Lambda_{\rm s} \equiv \Lambda_{\rm s0}\,{\rm sgn}[z_\dagger - z]$



Phantom scalar field !!!

$$S_\phi = \int d^4x\sqrt{-g}\left[\frac{\xi}{2}g^{ik}\partial_i\phi\partial_k\phi - V(\phi)\right]$$

$\xi = -1$ - **phantom**

$\xi = +1$ - **quinessence**

Scalar field energy density and pressure:

$$\varepsilon_\phi \equiv T^0_0(\phi) = \frac{\xi}{2c^2}(\dot{\phi})^2 + V(\phi), \qquad p_\phi \equiv -T^\alpha_\alpha(\phi) = \frac{\xi}{2c^2}(\dot{\phi})^2 - V(\phi),$$

Equation of motion:

$$\ddot{\phi} + 3H\dot{\phi} + \xi c^2\frac{dV}{d\phi} = 0$$

Equation of state parameter and speed of sound squared:

$$\omega_\phi = \frac{p_\phi}{\varepsilon_\phi} = \frac{\xi X - V(\phi)}{\xi X + V(\phi)}, \quad X \equiv \frac{1}{2c^2}(\dot{\phi})^2 \qquad c_s^2 = \frac{\partial p_\phi}{\partial \varepsilon_\phi}c^2 = \frac{p_{\phi},_X}{\varepsilon_{\phi},_X}c^2 = \frac{\xi}{\xi}c^2 = c^2$$

Two phantom branches:

n-phantom: $\varepsilon_\phi = -X + V < 0 \quad \Rightarrow \quad \omega_\phi > -1$

p-phantom: $\varepsilon_\phi = -X + V > 0 \quad \Rightarrow \quad \omega_\phi < -1$

n-phantom to p-phantom crossing:

For both brances!

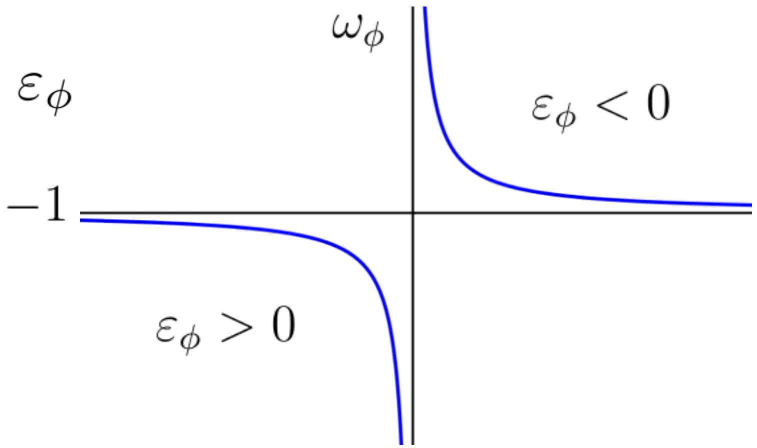
Continuity equation:

$$\dot{\varepsilon}_\phi \; = \; -3H\left(\varepsilon_\phi + p_\phi\right) = -3H\,\varepsilon_\phi\left(1 + \omega_\phi\right) > 0$$

$$\left.\begin{array}{lll} \varepsilon_\phi \longrightarrow 0^- & \Rightarrow & \omega_\phi \longrightarrow +\infty \\ \varepsilon_\phi \longrightarrow 0^+ & \Rightarrow & \omega_\phi \longrightarrow -\infty \end{array}\right\}$$

ω_ϕ **has a pole because of** ε_ϕ

safe singularity!



$$\dot{\varepsilon}_\phi = 6HX \quad \text{for} \quad \varepsilon_\phi = 0$$

ε_ϕ **can smoothly approach zero from below, crossing** $\varepsilon_\phi = 0$ **in finite time without any singular behavior!**

However, possible problems:

- Violation of the null energy condition $(p+\varepsilon<0)$.
- Energy density may be negative $(\varepsilon<0)$.
- Big rip singularity $(a, \varepsilon \longrightarrow \infty$ for finite $t)$.
- Instability of the model for unbounded potentials $V(\phi) \longrightarrow \infty$.

violation of WEC

?

We shall demonstrate that:

the addition of ordinary matter (e.g. CDM + radiation) and the appropriate choice of potential energy V allows us to circumvent these problems.

!

Potential:

$$V(\phi) = \frac{\Lambda(\xi_1+1)}{2} - \frac{\Lambda(\xi_1-1)}{2}\tanh\left[\sqrt{\kappa}\nu(\phi-\phi_c)\right] \quad , \quad \Lambda > 0$$

dictates the rapidity of transition

Asymptotic behavior:

$$V(\phi\to-\infty)\to\Lambda\xi_1 \quad , \qquad V(\phi\to+\infty)\to\Lambda$$

- $\xi_1 = 1 \quad \Longrightarrow \quad$ standard ΛCDM model;
- $\xi_1 < 0,\ \xi = -1(\text{phantom!}) \quad \Longrightarrow \quad$ AdS-dS transition;
- $\xi_1 = 0,\ \xi = -1(\text{phantom!}) \quad \Longrightarrow \quad$ 0-dS transition (emerging cosmological constant);
- $\xi_1 > 0, \quad \Longrightarrow \quad$ dS-dS transition;
  1) $\xi_1 < 1,\ \xi = -1$(phantom!) scalar field evolves up its potential;
  2) $\xi_1 > 1,\ \xi = +1$(quintessence!) scalar field evolves down its potential;

Background model

FLRW-metric:

$$ds^2 = c^2 dt^2 - a^2 \delta_{\alpha\beta} dx^\alpha dx^\beta$$

CDM and radiation:

$$\varepsilon_{\rm m} = \varepsilon_{\rm m0}/a^3, \qquad \varepsilon_{\rm r} = \varepsilon_{\rm r0}/a^4\,, \qquad a_0 = 1 \qquad (1)$$

Scalar field

$$\varepsilon_\phi \equiv T_0^0(\phi) = \frac{\xi}{2c^2}(\dot\phi)^2 + V(\phi)\,, \qquad p_\phi \equiv -T_\alpha^\alpha(\phi) = \frac{\xi}{2c^2}(\dot\phi)^2 - V(\phi); \qquad (2)$$

Equations of motion:

$$\frac{3H^2}{c^2} = \kappa\left(\varepsilon_{\rm m} + \varepsilon_{\rm r} + \varepsilon_\phi\right)\,, \qquad (3)$$

$$\frac{2\dot H + 3H^2}{c^2} = -\kappa\left(p_{\rm r} + p_\phi\right)\,, \qquad (4)$$

$$\ddot\phi + 3H\dot\phi + \xi c^2 \frac{dV}{d\phi} = 0; \qquad (5)$$

It is convenient to use the redshift instead of synhronous time and introduce the dimensionless parameters:

$$z = 1/a - 1 \qquad \Longrightarrow \qquad \mathrm{d}/\mathrm{d}t = -(1+z)H\mathrm{d}/\mathrm{d}z.$$

$$\Omega_{\mathrm{m}0} \equiv \frac{\varepsilon_{\mathrm{m}0}}{\varepsilon_{\mathrm{cr}0}}, \; \Omega_{\mathrm{r}0} \equiv \frac{\varepsilon_{\mathrm{r}0}}{\varepsilon_{\mathrm{cr}0}}, \; \Omega_{\Lambda 0} \equiv \frac{\Lambda}{\varepsilon_{\mathrm{cr}0}},$$

$$\widetilde{\Omega}_{\phi}(z) \equiv \frac{\varepsilon_{\phi}}{\varepsilon_{\mathrm{cr}0}}, \; \widetilde{h}(z) \equiv \frac{H}{H_0}, \; \tilde{\phi}(z) \equiv \sqrt{\kappa}\phi,$$

$$\widetilde{\Omega}_{K}(z) \equiv \frac{\xi}{2c^2}\frac{(\dot{\phi})^2}{\varepsilon_{\mathrm{cr}0}} = \frac{\xi}{6}(1+z)^2\widetilde{h}^2\left(\frac{\mathrm{d}\widetilde{\phi}}{\mathrm{d}z}\right)^2,$$

$$\widetilde{\Omega}_{V}(z) \equiv \frac{V}{\varepsilon_{\mathrm{cr}0}} = \Omega_{\Lambda 0}\left\{\frac{\xi_1+1}{2} - \frac{\xi_1-1}{2}\tanh\left[\nu\left(\tilde{\phi}-\tilde{\phi}_{\mathrm{c}}\right)\right]\right\}$$

$$\varepsilon_{\mathrm{cr}0} = 3H_0^2/\kappa c^2 \qquad\qquad H_0 = H(z=0)$$

Equations (1)-(5):

(1) $\Rightarrow$ $$\widetilde{\Omega}_{\mathrm{m}}(z) = \Omega_{\mathrm{m}0}(1+z)^3\,, \quad \widetilde{\Omega}_{\mathrm{r}}(z) = \Omega_{\mathrm{r}0}(1+z)^4\,,$$

(2) $\Rightarrow$ $$\frac{p_\phi}{\varepsilon_{\mathrm{cr}0}} = \widetilde{\Omega}_K - \widetilde{\Omega}_V\,, \qquad \widetilde{\Omega}_\phi = \widetilde{\Omega}_K + \widetilde{\Omega}_V\,,$$

(3), (4) $\Rightarrow$ $$\widetilde{h}^2 = \widetilde{\Omega}_{\mathrm{m}} + \widetilde{\Omega}_{\mathrm{r}} + \widetilde{\Omega}_\phi\,, \qquad 2(1+z)\widetilde{h}\frac{\mathrm{d}\widetilde{h}}{\mathrm{d}z} = 3\widetilde{\Omega}_{\mathrm{m}} + 4\widetilde{\Omega}_{\mathrm{r}} + 6\widetilde{\Omega}_K$$

(5) $\Rightarrow$ $$(1+z)^2\widetilde{h}^2\frac{\mathrm{d}^2\widetilde{\phi}}{\mathrm{d}z^2} + (1+z)^2\widetilde{h}\frac{\mathrm{d}\widetilde{h}}{\mathrm{d}z}\frac{\mathrm{d}\widetilde{\phi}}{\mathrm{d}z} - 2(1+z)\widetilde{h}^2\frac{\mathrm{d}\widetilde{\phi}}{\mathrm{d}z} + 3\xi\frac{\mathrm{d}\widetilde{\Omega}_V}{\mathrm{d}\widetilde{\phi}} = 0$$

$$\Omega_{\mathrm{m,r}} \equiv \varepsilon_{\mathrm{m,r}}/\varepsilon_{\mathrm{cr}} = \widetilde{\Omega}_{\mathrm{m,r}}/\widetilde{h}^2$$

Additional useful expressions:

Scalar field EoS:

$$\omega_\phi = \frac{p_\phi}{\varepsilon_\phi} = \frac{\widetilde{\Omega}_K - \widetilde{\Omega}_V}{\widetilde{\Omega}_K + \widetilde{\Omega}_V}$$

EoS for total matter content:

$$\omega_{\rm tot} = \frac{p_{\rm tot}}{\varepsilon_{\rm tot}} = \frac{p_{\rm m} + p_{\rm r} + p_\phi}{\varepsilon_{\rm m} + \varepsilon_{\rm r} + \varepsilon_\phi} = \left[\frac{1}{3}\widetilde{\Omega}_{\rm r} + \widetilde{\Omega}_K - \widetilde{\Omega}_V\right] / \left[\widetilde{\Omega}_{\rm m} + \widetilde{\Omega}_{\rm r} + \widetilde{\Omega}_K + \widetilde{\Omega}_V\right]$$

Dimensionless total energy density:

$$\widetilde{\Omega}_{\rm tot}(z) = \frac{\varepsilon_{\rm tot}}{\varepsilon_{\rm cr0}} = \widetilde{\Omega}_{\rm m} + \widetilde{\Omega}_{\rm r} + \widetilde{\Omega}_K + \widetilde{\Omega}_V$$

Hubble parameter derivative:

$$\frac{\dot{H}}{H_0^2} = -\frac{1}{2}\left(3\widetilde{\Omega}_{\rm m} + 4\widetilde{\Omega}_{\rm r} + 6\widetilde{\Omega}_K\right)$$

Deceleration parameter:

$$q = -1 + \frac{\rm d}{{\rm d}t}\frac{1}{H} = \frac{\frac{1}{2}\widetilde{\Omega}_{\rm m} + \widetilde{\Omega}_{\rm r} + 2\widetilde{\Omega}_K - \widetilde{\Omega}_V}{\widetilde{\Omega}_{\rm m} + \widetilde{\Omega}_{\rm r} + \widetilde{\Omega}_K + \widetilde{\Omega}_V} .$$

Numerical integration

We assume that prerecombination universe is well described by the ΛCDM model, and we start calculations from the redshift $z_* \sim 1090$ of last scattering.

ΛCDM → comoving sound horizon: $r_* = \int_{z_*}^{\infty} c_s H(z)^{-1} dz,$

Planck CMB spectra → angular scale of the sound horizon: $\theta_* = r_*/D_M(z_*);$

Comoving angular diameter distance to last scattering:

$$D_M(z_*) = 13869.57\,\mathrm{Mpc};$$

Planck CMB spectra → $\Omega_{m0}h^2 = 0.14314$ (*) $h \equiv H_0/(100\,\mathrm{km\,s^{-1}\,Mpc^{-1}})$

We took into account contribution from both photons and neutrinos

$$\Omega_{r0}\, h^2 = 2.469 \times 10^{-5} \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\mathrm{eff}}\right] \quad (**)$$

$N_{\mathrm{eff}} = 3.046$

is determined by CMB monopole temperature

For our phantom model with $H = H(z)$, we fix the angular diameter distance to the last scattering surface:

$$\mathrm{D}_M(z_*) = \int_0^{z_*} c\frac{dz}{H} = \frac{c}{H_0}\int_0^{z_*}\frac{dz}{\tilde{h}} = 13872.83\,\mathrm{Mpc}\,. \qquad (***)$$

$\boldsymbol{H_0}$ **is selected in accordance with the SH0ES data:** $H_0 = 73.04\,\mathrm{km\,s^{-1}Mpc^{-1}}$

➡ **This automatically determines the transition redshift value $z_\dagger$.**

For the given $\boldsymbol{H_0}$, we obtain $\Omega_{\mathrm{m}0}$ and $\Omega_{\mathrm{r}0}$.

$$\widetilde{\Omega}_\phi(z=0) \equiv \Omega_{\phi 0} = 1 - \Omega_{\mathrm{m}0} - \Omega_{\mathrm{r}0} \qquad \Omega_{V0} \approx \Omega_{\Lambda 0} \approx \Omega_{\phi 0}\,.$$

Initial conditions: $\tilde{\phi}_{\mathrm{in}} \equiv \tilde{\phi}(z = z_*) = 0, \quad \tilde{\phi}'_{\mathrm{in}} \equiv \dfrac{d\tilde{\phi}}{dz}(z = z_*) = 0$

Rapidity parameter: $\nu = 100.$

Scenario:

$$D_M(z_*) = \int_0^{z_*} c\frac{dz}{H} = \frac{c}{H_0}\int_0^{z_*}\frac{dz}{\tilde{h}} = 13872.83\,\mathrm{Mpc}\,. \quad (***)$$

Any suppression of $H(z)$ (i.e. $H(z) < H(z)_{\Lambda CDM}$) for $z > z_\dagger$ due to the negative phantom energy density, must be compensate by an enhancement ($H(z) > H(z)_{\Lambda CDM}$) at lower redshifts, $z > z_\dagger$, to maintain consistency with (***).

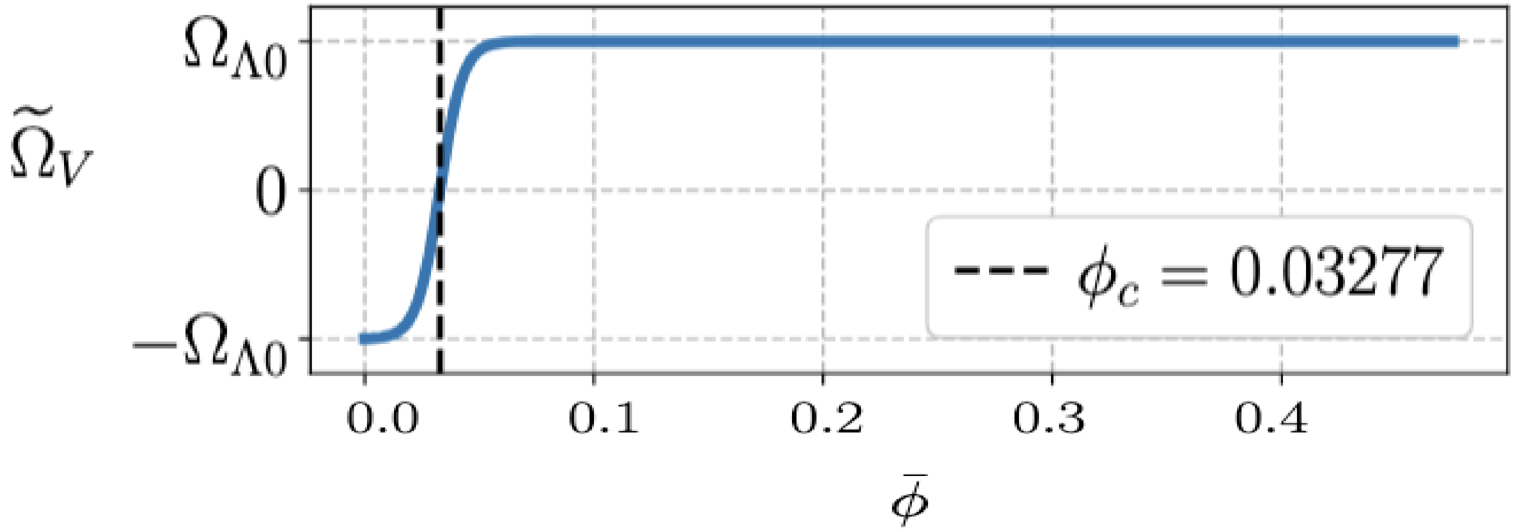
This implies an increesed $H_0 \equiv H(z=0)$ and thereby also a decreased $\boldsymbol{\Omega_{mo}}$ compared to the Planck- ΛCDM model.

Mirror $\mathrm{AdS} \longrightarrow \mathrm{dS}$ transition : $\Lambda_{\mathrm{s}}\mathrm{CDM}$

$$\xi = -1, \ \xi_1 = -1$$

Potential:

$$V(\tilde{\phi}) = \Lambda \tanh\left[\nu(\tilde{\phi} - \tilde{\phi}_{\mathrm{c}})\right], \quad \Lambda > 0 \quad \Longrightarrow \quad \widetilde{\Omega}_V = \Omega_{\Lambda 0} \tanh[\nu(\tilde{\phi} - \tilde{\phi}_{\mathrm{c}})]$$



$$\widetilde{\Omega}_V(z) \equiv \frac{V}{\varepsilon_{\mathrm{cr}0}}$$

$$\Omega_{\Lambda 0} \equiv \frac{\Lambda}{\varepsilon_{\mathrm{cr}0}}$$

For chosen values of ν **and** H_0

$z_{\mathrm{t}} \approx 2.12 \qquad \widetilde{\Omega}_V(z = z_{\mathrm{t}}) = 0 \qquad z_{\dagger} \approx 1.79 \qquad \widetilde{\Omega}_\phi(z = z_{\dagger}) = 0$

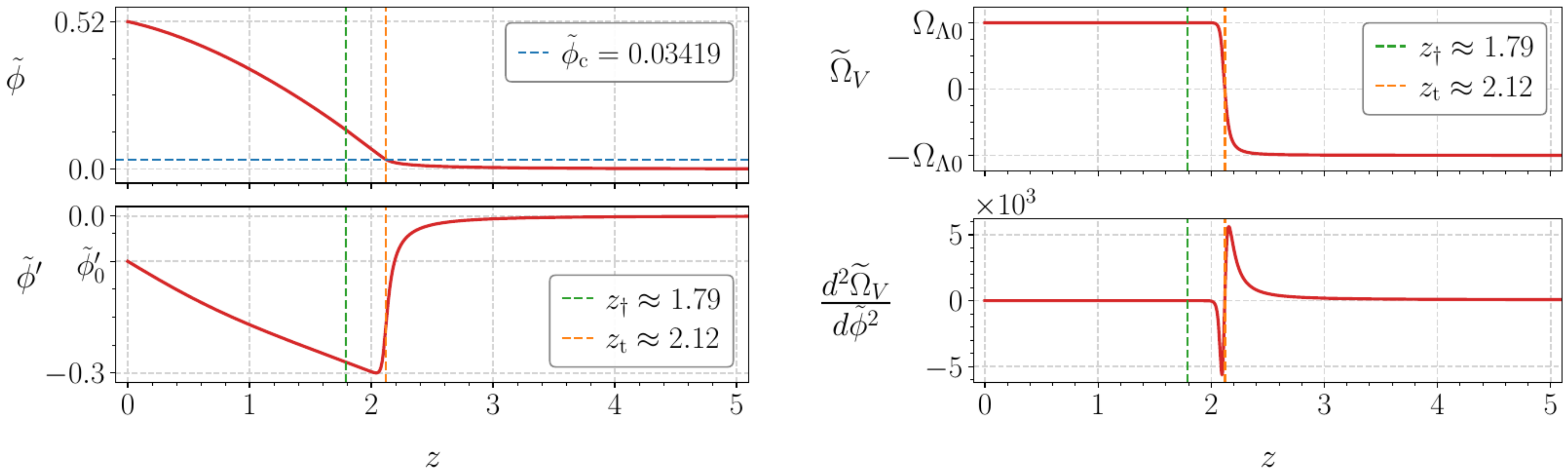
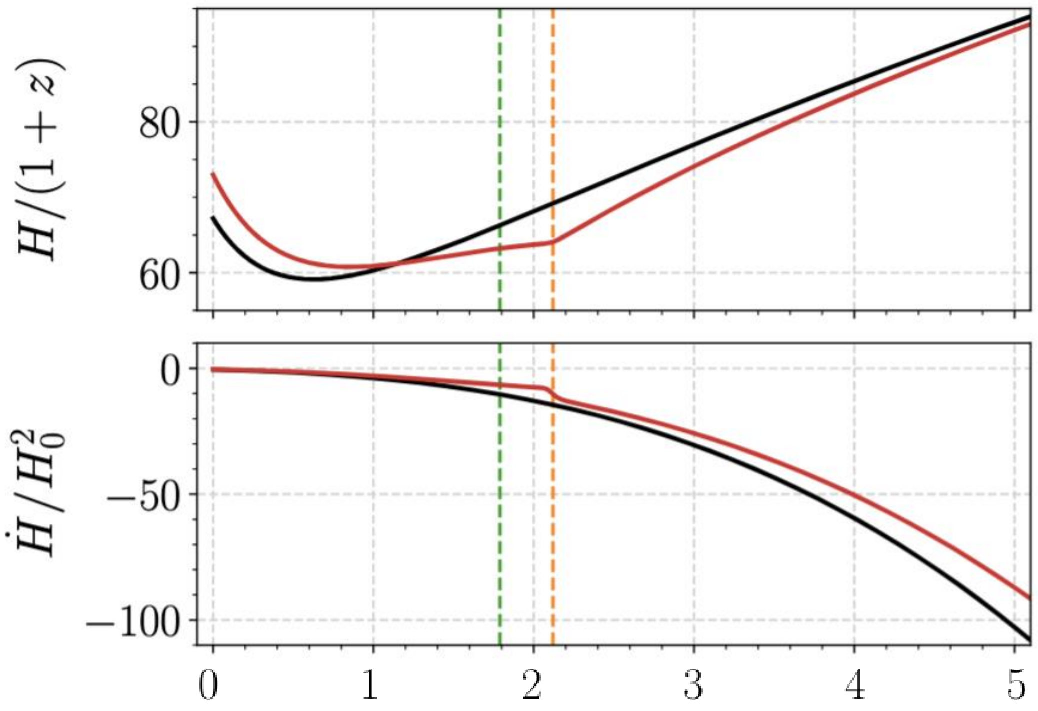
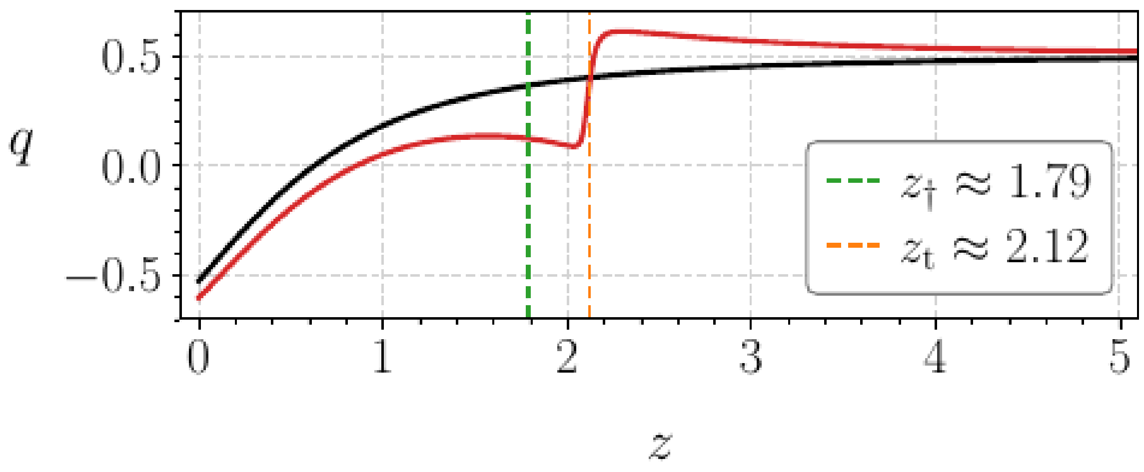


FIG. 4. On the left panel: The dimensionless phantom field $\tilde{\phi}$ and its derivative $\tilde{\phi}' \equiv d\tilde{\phi}/dz$. The value of this derivative at present time $\tilde{\phi}'_0 = -0.09661$. $\tilde{\phi}_c = 0.03419$ is the value of the phantom field where the potential changes sign. On the right panel: The dimensionless potential $\widetilde{\Omega}_V = V/\varepsilon_{cr0}$ and its second derivative where $\Omega_{\Lambda 0} = 0.73161$.





The Hubble parameter $H/(1+z)\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, the dimensionless combination $\dot{H}/(H_0^2)$, where $\dot{H} = dH/dt$, and the deceleration parameter q as functions of the redshift z. z_t determines the moment of the transition where the potential is equal to zero: $V(z = z_\mathrm{t}) = 0$ and $z_\dagger$ corresponds to the redshift at which the phantom energy density is equal to zero: $\widetilde{\Omega}_\phi(z = z_\dagger) = 0$. Black curves describe the standard ΛCDM model. For the present day Hubble parameter we get $H_0 = 74.03\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $H_0 = 67.22\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ for the phantom and ΛCDM models, respectively.

$$z_\mathrm{t} \gtrsim 2\left(\Omega_\mathrm{m0}^{-1} - 1\right)^{\frac{1}{3}} - 1 \Longrightarrow \dot{H} < 0 \quad \text{for} \quad 0 \le z \le +\infty \qquad \textbf{if} \quad \Omega_\mathrm{m0} \sim 0.27 \Longrightarrow z_\mathrm{t} \gtrsim 1.8.$$

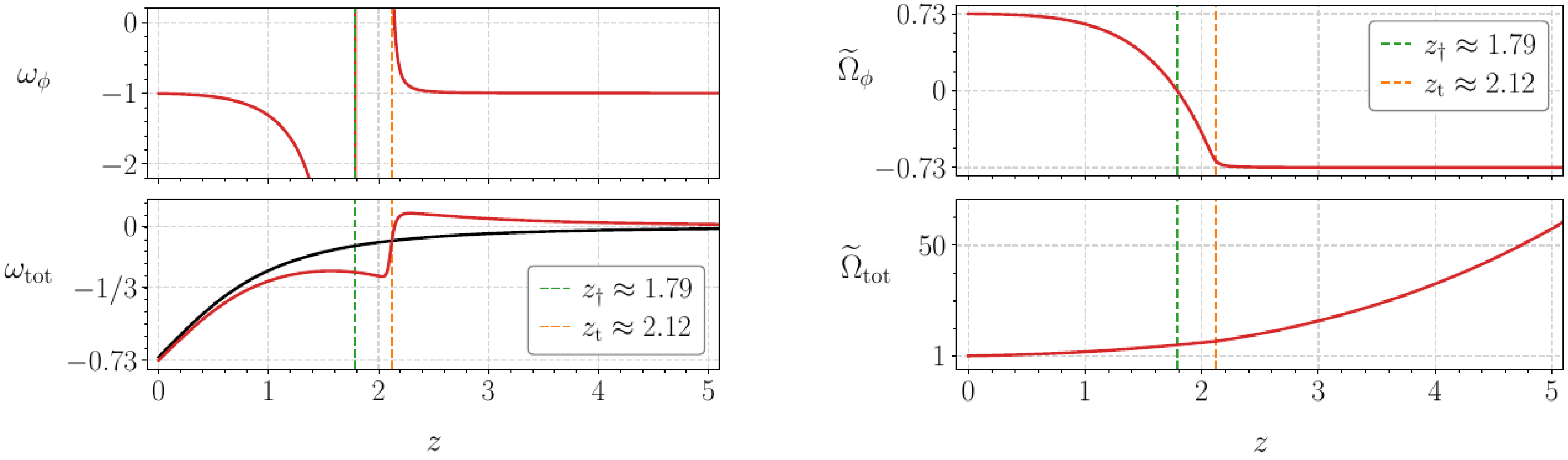


FIG. 5. On the left panel: the EoS parameter ω_ϕ for the phantom field and the EoS parameter $\omega_{\rm tot}$ for total matter (phantom + CDM + radiation). On the right panel: the dimensionless energy density $\widetilde{\Omega}_\phi = \varepsilon_\phi/\varepsilon_{\rm cr0}$ for the phantom and the dimensionless energy density $\widetilde{\Omega}_{\rm tot} = \varepsilon_{\rm tot}/\varepsilon_{\rm cr0}$ for total matter. The total energy density is everywhere positive.

$$\widetilde{\Omega}_{\rm tot}(z) = \frac{\varepsilon_{\rm tot}}{\varepsilon_{\rm cr0}} = \widetilde{\Omega}_{\rm m} + \widetilde{\Omega}_{\rm r} + \widetilde{\Omega}_\phi \geq 1 \quad \Rightarrow \quad \varepsilon_{\rm tot} > 0$$

$$\omega_{\rm tot} = \frac{p_{\rm tot}}{\varepsilon_{\rm tot}} = \frac{p_{\rm m} + p_{\rm r} + p_\phi}{\varepsilon_{\rm m} + \varepsilon_{\rm r} + \varepsilon_\phi} > -1$$

$$p_{\rm tot} + \varepsilon_{\rm tot} > 0$$

WEC is satisfied!

General transition models

$$V(\phi) = \frac{\Lambda(\xi_1+1)}{2} - \frac{\Lambda(\xi_1-1)}{2} \tanh\left[\sqrt{\kappa}\nu\,(\phi-\phi_c)\right] \quad , \quad \Lambda > 0$$

Asymptotic behavior:

$$V(\phi \to -\infty) \to \Lambda\xi_1 \quad , \qquad V(\phi \to +\infty) \to \Lambda$$

- $\xi = 0,\ \xi_1 = 1 \quad \Longrightarrow$ standard ΛCDM model;
- $\xi = -1,\ \xi_1 = -1$ (phantom!) $\Longrightarrow$ AdS-dS transition;
- $\xi = -1,\ \xi_1 = -0.5$(phantom!) $\Longrightarrow$ AdS-dS transition;
- $\xi = -1,\ \xi_1 = 0$ (phantom!) $\Longrightarrow$ 0-dS transition (emerging CC);
- $\xi = -1,\ \xi_1 = 0.5$ (phantom!) $\Longrightarrow$ dS-dS transition;
- $\xi = 1,\ \ \xi_1 = 1.5$(quintessence!) $\Longrightarrow$ dS-dS transition;

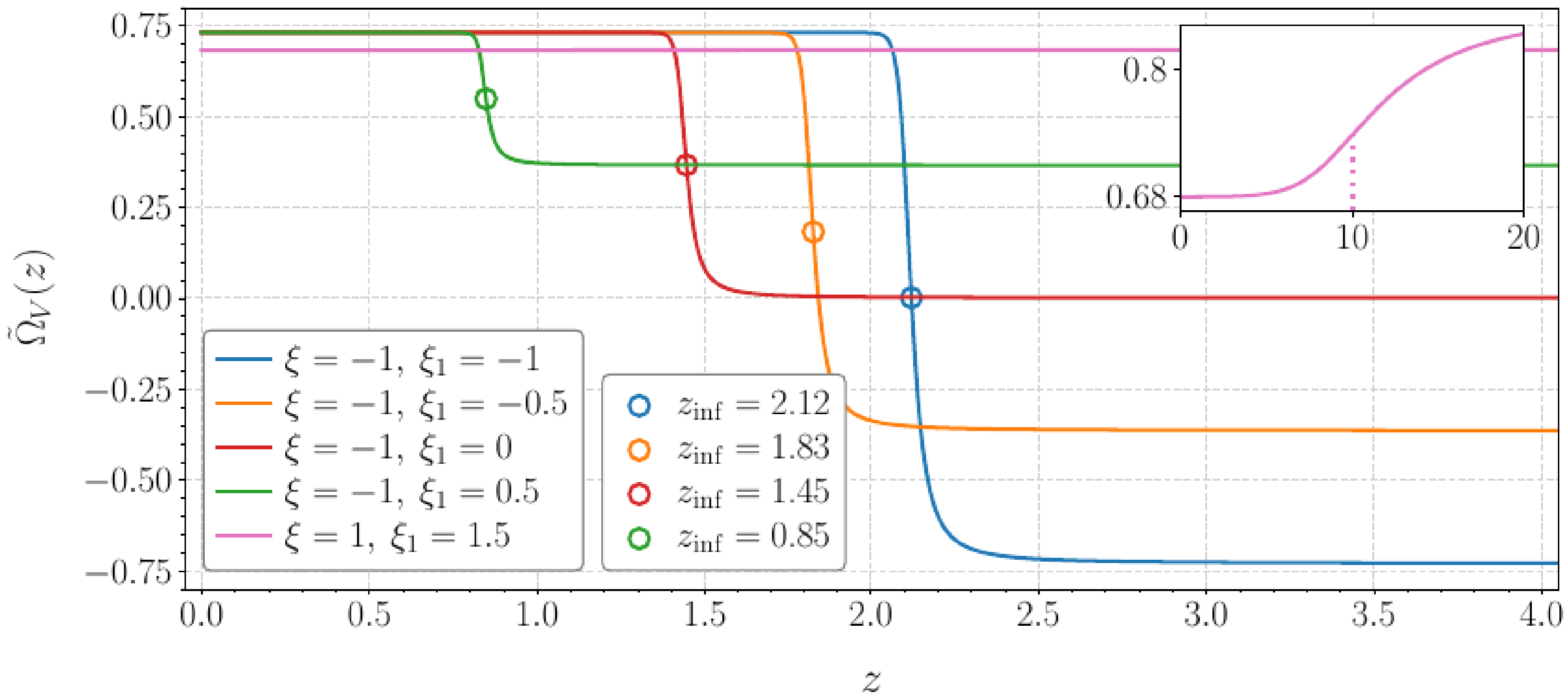


FIG. 6. The dimensionless potential energy of the phantom ($\xi = -1$) with parameters $\xi_1 = -1, -0.5, 0, 0.5$ and quintessence ($\xi = 1$) with $\xi_1 = 1.5$. The marked dots highlight the inflection points of the phantom potentials. Potentials changes sharply at the transition for the phantom. We zoomed a region where quintessence undergoes a smooth transition with the inflaction point at $z \sim 10$.

$z_{\rm inf}$ **are the inflection points of the potential** $\widetilde{\Omega}_V$;

it characterizes the moment of transition

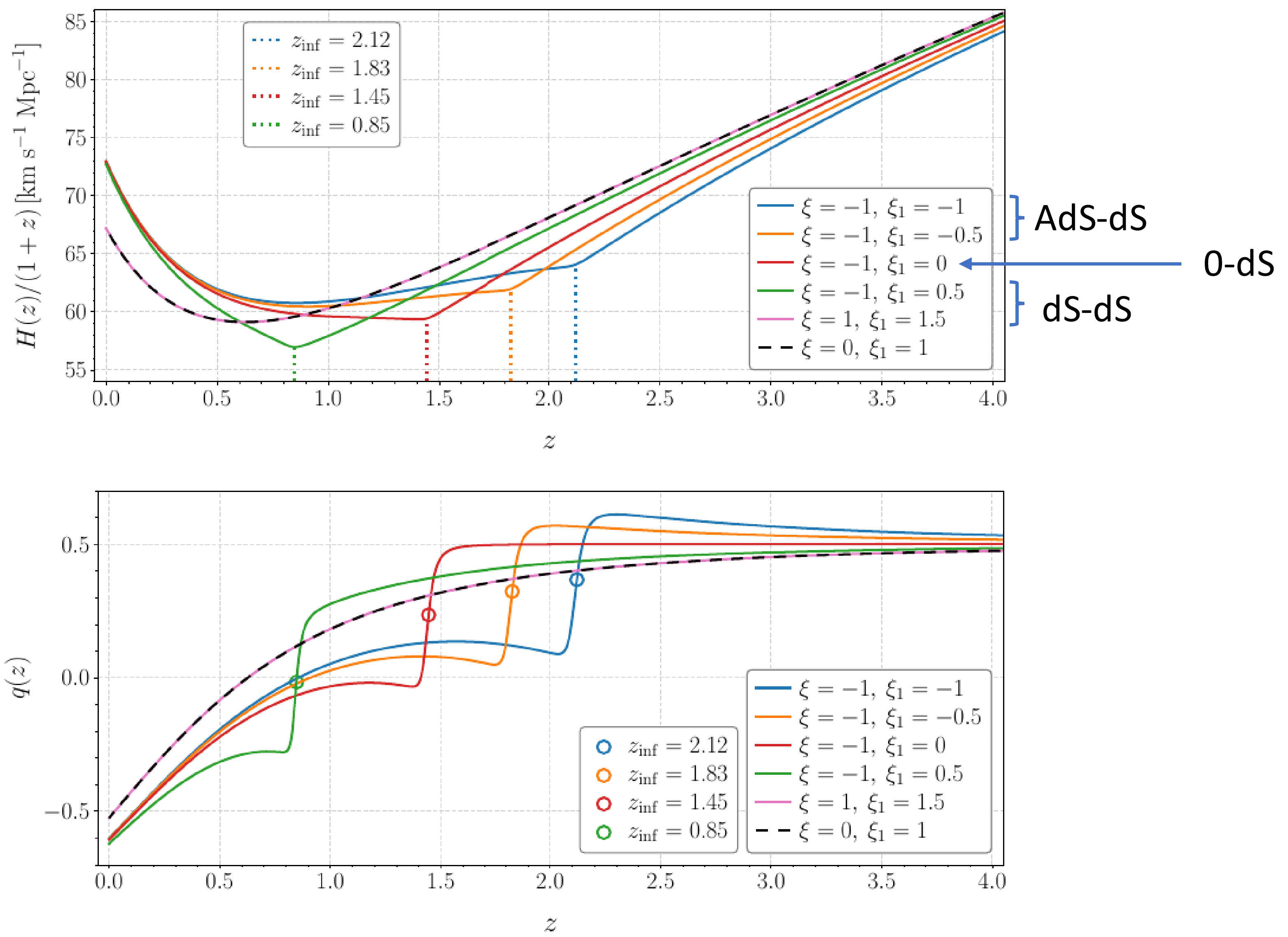


FIG. 9. Upper panel: The evolution of the co-moving Hubble parameter, $\dot{a} = H(z)/(1+z)$, for selected models. All curves are continuous and the inflection points of $\widetilde{\Omega}_V$ define the characteristic points of $H(z)$. The phantom cases, i.e. $\xi < 0$, satisfy the present-day value for the Hubble parameter in accordance with the SH0ES, that is 73.04. The black dashed line corresponds to ΛCDM solution which gives $H_0 = 67.22$ whereas for the quintessence case represented by pink curve $H_0 = 67.21$ is obtained. Lower panel: The deceleration parameters for the selected models. All curves are continuous and smooth and in the case of the phantom demonstrate the rapid transition. The inflection points of these transitions occur at the same redshifts as the inflection points for the corresponding potentials $\widetilde{\Omega}_V$.

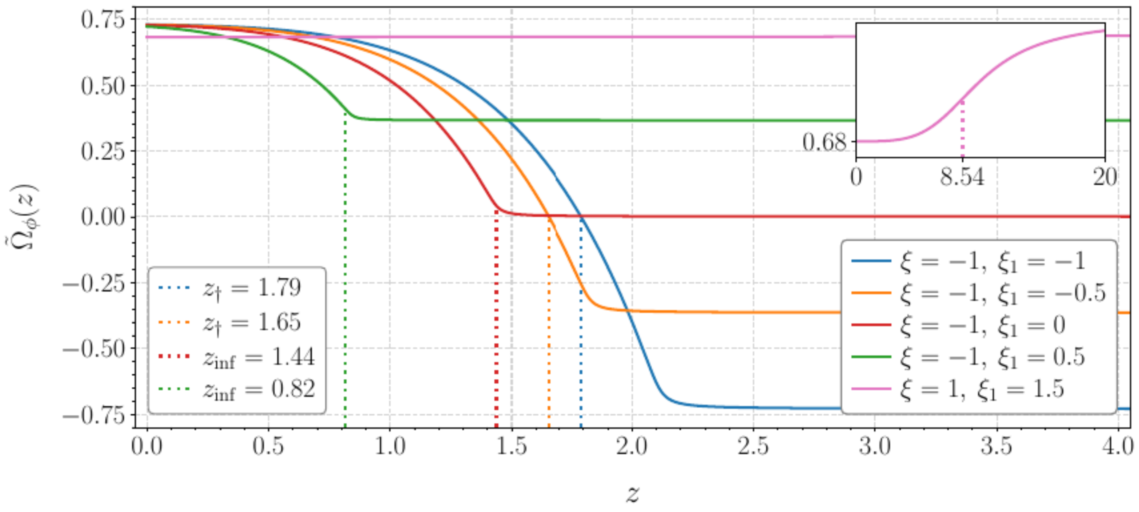


FIG. 7. The dimensionless total energy of scalar field. Redshifts $z = 0.82, 1.44, 8.54$ correspond to the inflection points of $\widetilde{\Omega}_\phi(z)$ for green, red and pink curves whereas $z = 1.65, 1.79$ indicates times when $\widetilde{\Omega}_\phi(z)$ crosses zero for orange and blue curves, respectively.

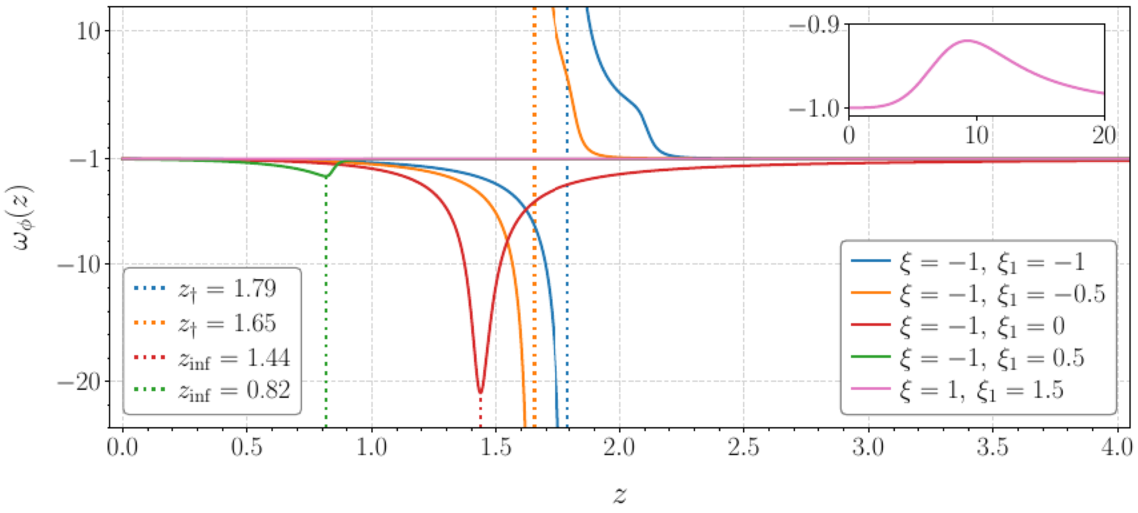


FIG. 8. The behavior of the equation of state parameters ω_ϕ is defined by the characteristic points of the scalar field total energy $\widetilde{\Omega}_\phi$. In the phantom cases with $\xi_1 \geq 0$, the inflection points of $\widetilde{\Omega}_\phi$ correspond to sharp (but smooth) minima of ω_ϕ. If the total phantom energy crosses zero (that happens for $\xi_1 < 0$ in the case of blue and orange lines), then the equation of state parameter diverges at this redshift.

Perturbation analysis

Perturbed scalar field:

$$\phi = \phi_b(\eta) + \varphi(\eta, \mathbf{r})$$

Equation of motion:

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{c^2}{a^2}\Delta\varphi - c^2\delta\frac{dV}{d\phi} = 0.$$

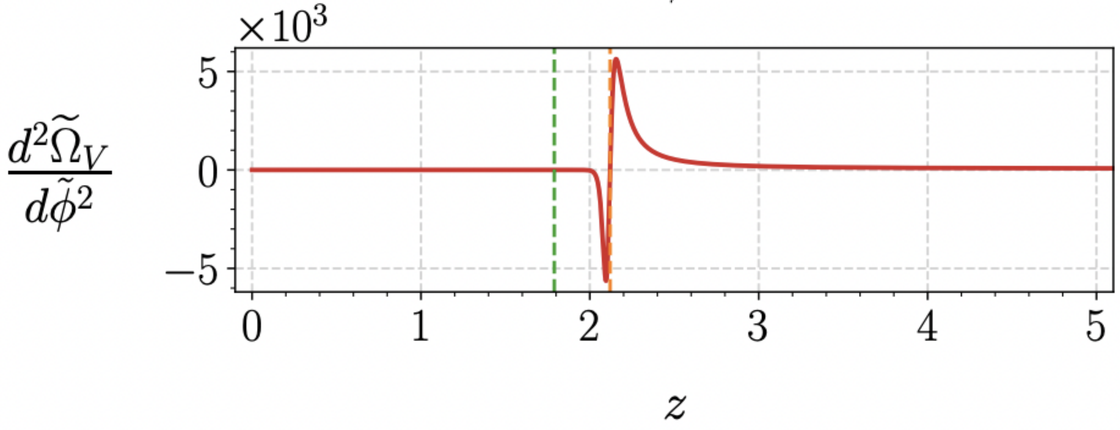
Fourier transformation $\Big\downarrow$ Dimnsionless units

$$(1+z)^2\tilde{h}^2\frac{d^2\tilde{\varphi}}{dz^2} + (1+z)^2\tilde{h}\frac{d\tilde{h}}{dz}\frac{d\tilde{\varphi}}{dz} - 2(1+z)\tilde{h}^2\frac{d\tilde{\varphi}}{dz} + \underbrace{\left[\frac{c^2}{H_0^2}(1+z)^2k^2 - 3\frac{d^2\tilde{\Omega}_V}{d\tilde{\phi}^2}|_{\tilde{\phi}_b}\right]}_{m_{\rm eff}^2(z)}\tilde{\varphi} = 0$$

Effective mass squared:

$$m_{\rm eff}^2(z) = \frac{c^2}{H_0^2}(1+z)^2k^2 - 3\frac{d^2\tilde{\Omega}_V}{d\tilde{\phi}^2}|_{\tilde{\phi}_b}$$

$$\tilde{\Omega}_V = \Omega_{\Lambda 0}\tanh[\nu(\tilde{\phi}-\tilde{\phi}_c)] \quad \Longrightarrow \quad \frac{d^2\tilde{\Omega}_V}{d\tilde{\phi}^2}|_{\tilde{\phi}_b} = -2\Omega_{\Lambda 0}\nu^2 \mathrm{sech}^2[\nu(\tilde{\phi}_b-\tilde{\phi}_c)]\tanh[\nu(\tilde{\phi}_b-\tilde{\phi}_c)]$$



$$\boxed{k^2 < \frac{3H_0^2}{c^2}(1+z)^{-2}\frac{d^2\tilde{\Omega}_V}{d\tilde{\phi}^2}|_{\tilde{\phi}_b} \quad \Longrightarrow \quad m_{\rm eff}^2(z) < 0 \quad !}$$

$$k_{\rm max} = \frac{2H_0\nu}{c(1+z)}\sqrt{\frac{\Omega_{\Lambda 0}}{\sqrt{3}}}. \approx \frac{\nu}{z+1}\frac{H_0}{c}1.2998 \approx \frac{\nu}{z+1}1.0262\times 10^{-28}\,\mathrm{cm}^{-1}$$

$$H_0 = 73.04\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \approx 2.3668\times 10^{-18}\mathrm{s}^{-1},\ \Omega_{\Lambda 0} = 0.73161$$

Numerical calculations:

$k < k_{\mathrm{max}}$

$k > k_{\mathrm{max}}$

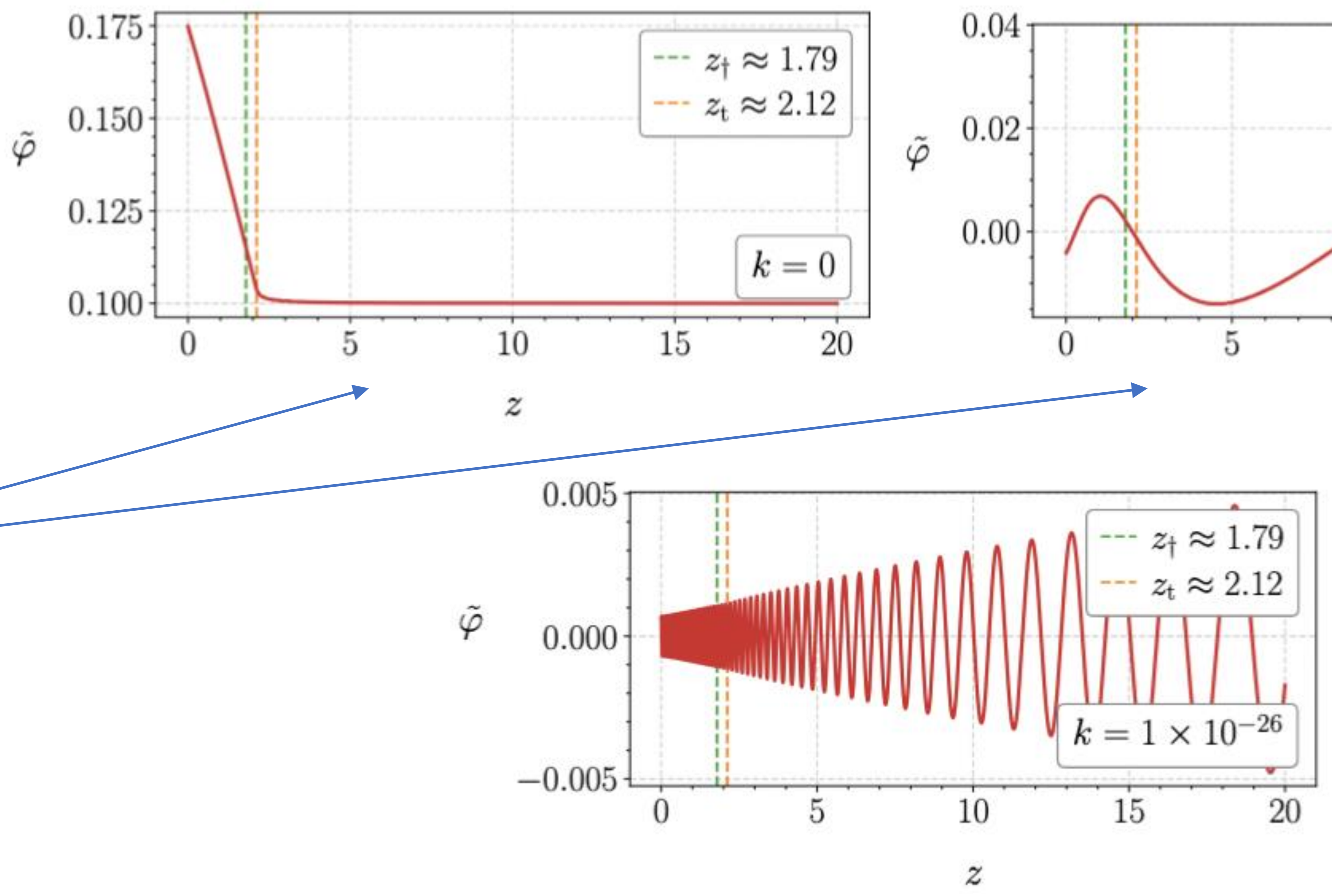
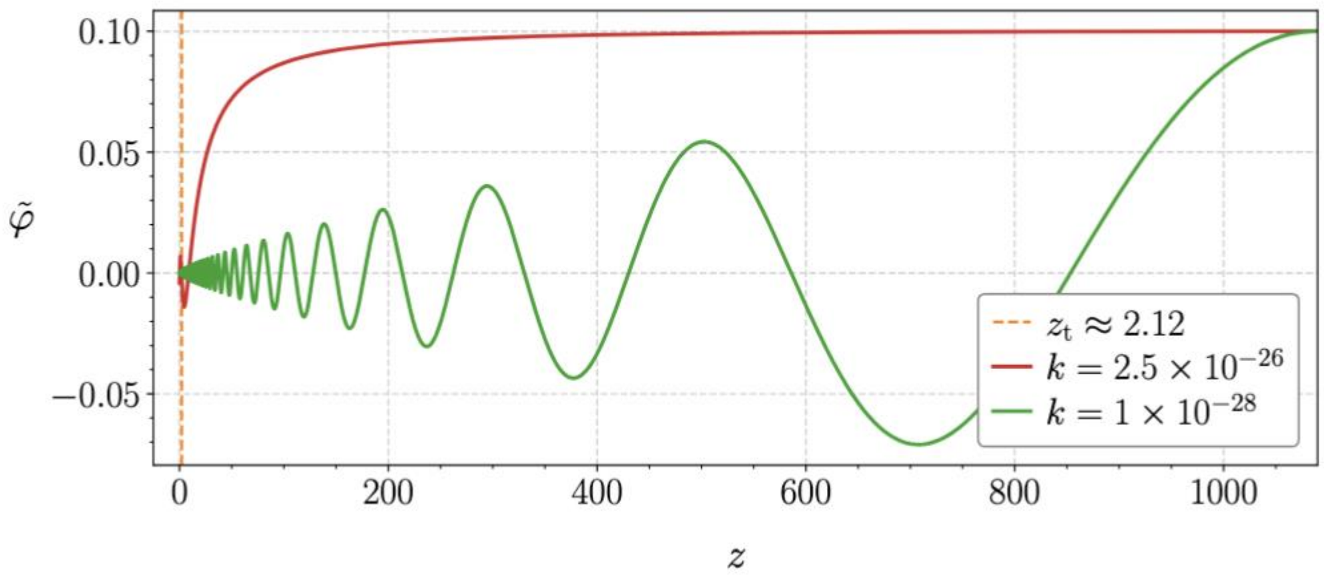


Figure 1: Evolution of scalar field perturbations $\tilde{\varphi}(z)$ for $\nu = 100$ and selected wavenumbers. The vertical dashed lines indicate the transition redshift $z_t = 2.12$ (orange line) and redshift $z_\dagger$ of the $\varepsilon_\phi = 0$ crossing (green line). Initial values have been taken as $\tilde{\varphi}_{\mathrm{in}} = 0.1$ and $\tilde{\varphi}'_{\mathrm{in}} = 0$.



1. There is no pathological growth of perturbations during $\varepsilon_\phi = 0$ crossing.

2. Initially small perturbations remain small.

⬇

Phantom field perturbations are stable!

Conclusions

- **Based on the theory of a phantom field with a hyperbolic tangent potential, we present a physical justification for the ΛsCDM model: Ph- ΛsCDM model.**

- **Ph- ΛsCDM model is free from typical phantom field pathologies: unbounded energy growth, Big Rip singularities, violation of the weak energy condition.**

- **The numerical integration of the equations of motion is performed ensuring consistency with the Planck CMB power spectra (fixing the present-day physical matter density $\Omega_{\rm m0}\, h^2$ and the angular diameter distance to last scattering $D_M(z_*)$) and the local SH0ES measurement of H_0 , significantly reducing the problem of cosmological tensions.**

- **All physical parameters ($H, q, \varepsilon_{\phi,\mathrm{tot}}$) responsible for the dynamics of the universe are continuous and smooth with characteristic points corresponding to the moment of transition.**

- **Phantom models with non-symmetric AdS-dS and dS-dS transition are also viable.**

- **The quintessence models with dS-dS transition are not able to solve the H_0 tension problem.**

- **Phantom field perturbations are stable.**

What is next?

1. N-body simulations for a full set of equations for phantom and metric perturbations (Gevolution code). Power spectra for matter, metric and phantom perturbations, comparison with the standard ΛCDM model.

2. Constraints on model parameters based on observational data, likelihood analysis.

THANK YOU!